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Distributional change: Assessing the contribution of household income sources*

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Abstract: We develop a decomposition of distributional change by factor components to quantify how changes in the association between sources of income and changes in their marginal distributions contribute to the change in the distribution of household incomes over time. The two components are further broken down to isolate the contribution of specific income sources. Application to the change in the distribution of household incomes in Luxembourg between 2004 and 2013 reveals contrasted results: increased association between spouse earnings, public transfers, and taxes depressed the income share of poor households while changes in marginal distributions increased incomes in the upper half of the distribution.

Keywords: copula, dependence structure, factor decomposition, income sources, poverty and inequality.

JEL classification : C14; D31; D33.

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1 Introduction

Inspection of the evolution of income components—earnings, capital income or public transfers—is essential to understand changes in the distribution of household incomes. First, distinct components develop differently over time. For example, the literature on the functional distribution of income and related accounts of the changing shares of capital and labour incomes show how the personal distribution of income is shaped by different evolutions of factor prices (e.g., Glyn, 2009; Piketty, 2014; Atkinson and Lakner, 2017; Aaberge *et al.*, 2018). Second, households typically receive incomes from multiple sources. The correlation between sources of incomes can mitigate or re-inforce inequality. While public transfers, especially if means-tested, normally correlate negatively with market incomes and mitigate inequality in household income (e.g., Danziger *et al*., 1981), the correlation in the earnings of high-skill, double-income earners tend to re-inforce inequality through assortative mating (e.g, Greenwood *et al.*, 2014). Both the (marginal) distribution of income sources and their association matter to the final distribution of disposable household incomes that policy typically cares about.

This has long been recognized. Shorrocks (1982) and Lerman and Yitzhaki (1985) have proposed early index decomposition approaches to uncover the contribution of income sources to total income inequality[1]. Extensions of this method have been used to explain changes in income inequality over time (e.g., Fiorio, 2011; García-Peñalosa and Orgiazzi, 2013; Brewer and Wren-Lewis, 2016). Similar decompositions have been developed to study other characteristics of income distributions such as indices of poverty (Mussard and Pi Alperin, 2011) or polarization (Deutsch *et al.*, 2013; Bárcena-Martin *et al.*, 2018; Bárcena-Martin and Silber, 2018). Decompositions of summary indices however put focus on particular distributional measures, which makes results dependent on the index of interest, e.g., the (square of) the coefficient of variation in the case of Shorrocks (1982) or the Gini coefficient for Lerman and Yitzhaki (1985). Furthermore, index decomposition approaches do not allow identifying "what

[1] See Chantreuil et al. (2019) for recent developments.

happens where” in the distribution. For example, the distribution might become more unequal because of the increased dispersion in its upper tail, lower tail or both, and such differences would have different implications for tentative policy actions.

To obtain a broader picture of the factors underlying distributional change over time, researchers turned to alternative decomposition techniques based on simulation of counterfactual distributions. For example, one can simulate the distribution of household incomes that would be observed today if, say, labour incomes had stayed put on past values. Comparisons of actual and such counterfactual distributions allow assessing the impact of changes in sources of income on the entire income distribution, freely from particular summary measures. Burtless (1999), notably, used a rank-preserving income exchange approach to evaluate, among other factors, how changes in the distributions of female and male earnings have contributed to the shift in the distribution of total income in the US between 1979 and 1996. In a similar spirit, Fournier (2001), Daly and Valletta (2006), Fiorio (2011) and Larrimore (2014) assessed the contributions of changes in various income sources to the inequality trends in Italy, Taiwan, and the US. These studies provide evidence on how shifts in distributions of income sources underlie changes in the distribution of household incomes. We develop this line of research here.

As hinted above, the overall distributive impact of changes in an income source—say, income from capital—depends on two factors: (i) the nature of the change in the source distribution itself and (ii) the (change in) association between the source of interest and the remaining household incomes. The first factor refers to the *marginal* distribution of the source—Is it growing in size? Is it becoming more or less unequally distributed? The second refers to the dependence structure of the various sources of household income—which are complementary, which are substitutes? Understanding the contribution of changes in income sources to aggregate distributional change requires sorting out the contribution of these factors. Only few studies have attempted to examine systematically how changes in the association between the income sources affected the overall distribution separately from the impact of the change in marginal distributions (see, e.g., Fournier, 2001; Larrimore 2014). Approaches differ across studies and the literature seems to lack a coherent analytical framework. The present paper attempts to address this concern by formalizing a general simulation-based, hierarchical decomposition procedure building upon copula theory. In a first step, the change in the distribution of total household incomes is apportioned into two components reflecting (1) changes in the marginal distributions of all income sources and (2) their dependence structure (as in Fournier, 2001). In a second step, the two aggregate components are decomposed further into contributions associated to each income source. Formalizing the decomposition in terms of marginal distributions and explicit copulas clarifies interpretation of its components and guides simulation strategies.

We apply the proposed methodology to the change in the household income distribution in Luxembourg between 2004 and 2013 during which the Gini index of inequality increased from 0.270 to 0.303 while relative income poverty rose by 2 percentage points (see e.g. Fusco *et al.*, 2014 or Allegrezza and Ametepe, 2018). Showing contrasted results along the income distribution, the analysis testifies of the relevance of the decomposition: increased association between spouse earnings, public transfers, and taxes depressed the income share of poor households while it is change in marginal distributions that drove changes in the upper half of the distribution.

2 Decomposing distributional change by income sources

Our overall strategy is to express the distribution of total income as a function of the joint distribution of income sources and to invoke Sklar's theorem (Sklar, 1959) to reformulate this distribution as a function of marginal distributions and of a copula capturing the dependence across sources. Having rewritten the distribution of income in terms of these components for two distinct time periods, we construct counterfactual distributions holding subsets of these components constant over time and build a sequential decomposition of the total change.

2.1 *The distribution of household income and its sources*

Consider two cross-sections of a population of N^t households observed in two time points $t \in \{0, 1\}$. At time $t \in \{0, 1\}$, each household $i \in \{1,\ldots,N^t\}$ receives income from different sources $j \in \{1,\ldots, k\}$ (e.g., earnings, capital income, public transfers etc.) so that

$$y_i^t = \sum_{j=1}^{k} y_{ij}^t \quad (1)$$

where y_i^t is the total income of household *i* in period *t* and y_{ij}^t is income from source *j*.[2]

To examine the contribution of multiple sources to total income, we start by expressing the CDF of total income in terms of the joint distribution of income components:

$$F^t(y) = \Pr[Y_1 + \ldots + Y_k \leq y \mid t] = \int_{-\infty}^{y} \int_{-\infty}^{y-y_1} \ldots \int_{-\infty}^{y-y_1-\ldots-y_{k-1}} dG^t(y_1,\ldots,y_k) \quad (2)$$

Equation (2) essentially integrates the joint density of all income sources over combinations that add up to a total income up to *y*.

Having an expression based on the joint distribution of sources, G^t, we invoke Sklar's (1959) fundamental theorem in the copula theory to express this joint distribution as a function of marginal (univariate) CDFs, F_j^t, and a copula $C_{1,\ldots,k}^t$ (Nelsen, 2006):

$$G^t(y_1,\ldots,y_k) = C_{1,\ldots,k}^t(F_1^t(y_1),\ldots,F_k^t(y_k)). \quad (3)$$

The copula is the joint CDF of *k* uniformly distributed variables ($r_1,\ldots, r_k$) where each variable contains information about the ranks of households in the marginal distributions of income components. The copula, thus, links the marginal distributions according to their rank dependence structure. The copula satisfying Equation (3) is unique for continuous marginal distributions, but not if any distribution has discrete components, as is typically the case for income sources which have a spike at zero. (We return to the implication of the non-uniqueness of the copula in Section 2.4.) Substituting Equation (3) into (2), the CDF of total household incomes can be re-expressed as a function of the copula and marginal distributions of income sources:

[2] Some of the sources, such as earnings, may be contributed by multiple members of the household. Some sources, such as taxes, may be negative (deducted from household incomes).

$$F^t(y) = \int_{-\infty}^{y} \int_{-\infty}^{y-y_1} \dots \int_{-\infty}^{y-y_1-\dots-y_{k-1}} dC^t_{1,\dots,k}(F^t_1(y_1), \dots, F^t_k(y_k)). \quad (4)$$

This copula-based representation of F^t provides us with a way to simulate counterfactual distributions that reflect changes arising from marginal distributions (through variations in any of the F^t_j) or from the dependence structure (through variations in $C^t_{1,\dots,k}$). In Section 2.2, we use this to spell out an aggregate 2-terms decomposition that quantifies the contributions of the copula and the marginal distributions. In Section 2.3, we further decompose each term into a new detailed (2k-1)-terms decomposition that quantifies the separate contribution of each separate source to both the copula and the marginal distributions components.

2.2 *Step 1: Aggregate decompositions*

Using Equation (4), the change in the distribution of household disposable incomes between time 0 and time 1 can be written

$$\Delta \mathrm{F(y)} = F^1(y) - F^0(y) = \int_{-\infty}^{y} \int_{-\infty}^{y-y_1} \dots \int_{-\infty}^{y-y_1-\dots-y_{k-1}} dC^1_{1,\dots,k}\left(F^1_1(y_1), \dots, F^1_{\mathrm{k}}(y_k)\right) -$$

$$\int_{-\infty}^{y} \int_{-\infty}^{y-y_1} \dots \int_{-\infty}^{y-y_1-\dots-y_{k-1}} dC^0_{1,\dots,k}\left(F^0_1(y_1), \dots, F^0_{\mathrm{k}}(y_k)\right). \quad (5)$$

Clearly, a change in the CDF can come about from changes in the copula, $C^t_{1,\dots,k}$, and/or from changes in the marginals $F^t_1, \dots, F^t_k$. To separate out these two sources, a counterfactual distributional change that captures the contribution of changes in the marginal distributions from period 0 to 1 holding the copula at reference period *c* can be constructed from Equation (5) as

$$\Delta F^{(c)}_M(y) = \int_{-\infty}^{y} \int_{-\infty}^{y-y_1} \dots \int_{-\infty}^{y-y_1-\dots-y_{k-1}} dC^{(c)}_{1,\dots,k}\left(F^1_1(y_1), \dots, F^1_k(y_k)\right) -$$

$$\int_{-\infty}^{y} \int_{-\infty}^{y-y_1} \dots \int_{-\infty}^{y-y_1-\dots-y_{k-1}} dC^{(c)}_{1,\dots,k}(F^0_1(y_1), \dots, F^0_{\mathrm{k}}(y_k)). \quad (6)$$

Similarly, a counterfactual distributional change that captures the contribution of the change in the copula holding the marginal distributions of all income sources fixed at reference period *m* can be constructed as

$$\Delta F_C^{(\mathrm{m})}(y) = \int_{-\infty}^{y} \int_{-\infty}^{y-y_1} \dots \int_{-\infty}^{y-y_1-\dots-y_{k-1}} dC_{1,\dots,k}^{1}(F_1^{(\mathrm{m})}(y_1), \dots, F_k^{(\mathrm{m})}(y_k)) -$$

$$\int_{-\infty}^{y} \int_{-\infty}^{y-y_1} \dots \int_{-\infty}^{y-y_1-\dots-y_{k-1}} dC_{1,\dots,k}^{0}\left(F_1^{(\mathrm{m})}(y_1), \dots, F_k^{(\mathrm{m})}(y_k)\right). \tag{7}$$

Combining (6) and (7), the total change from period 0 to period 1 can be additively decomposed as

$$\Delta F(y) = \Delta F_C^{(\mathrm{m})}(y) + \Delta F_M^{(\mathrm{c})}(y) \tag{8}$$

provided $(c, m) \in \{0,1\}^2$ and $c + m = 1$. The first condition imposes that the reference values for the copula and marginal distributions in the counterfactuals are those observed in either period 0 or period 1. The second condition is necessary for additive decomposability of the total change: it imposes that if the change in the copula is assessed holding marginal distributions at period 0 then the change in the marginal distributions is assessed holding the copula at period 1, or vice versa.[3] The term $\Delta F_C^{(\mathrm{m})}(y)$ can be interpreted as the contribution of the change in the copula to the overall income distribution change—that is, the change in the rank order association between income sources. The term $\Delta F_M^{(\mathrm{c})}(y)$ can be interpreted as the contribution of the change in marginal distributions.

Setting (c, m) to (0,1) or to (1,0) results in two alternative decompositions. Depending on application, one might opt for a "copula first" strategy identifying the contribution of the copula before the contribution of the marginal CDFs of income components and therefore choose (1,0), or the other way around. There is usually no strong justification for preferring one ("copula first") over the other ("marginal first") and estimation of the two is recommended. Combination of the decompositions can also be undertaken using a Shapley value approach (e.g., Sastre and Trannoy, 2002; Shorrocks, 2013).[4]

[3] Relaxing this restriction and maintaining additivity is possible but implies a third 'interaction' term whose interpretation is not immediate; see Biewen (2014).

[4] Chantreuil & Trannoy (2013) and Chantreuil et al. (2019) provide in-depth discussions of application of the Shapley value to decompositions of inequality functionals, including decompositions by income components. They combine related simulation approaches and the Shapley procedure, but focus on inequality functionals and on (static) contributions of sources to cross-sectional inequality.

The decomposition of the distribution change determines a decomposition of index functionals. Let Δθ denote the change in a generic index functional θ (e.g., the Gini coefficient):

$$\Delta\theta(F^0, F^1) = \theta(F^1) - \theta(F^0). \tag{9}$$

In the "copula first" approach, $(c, m) = (1,0)$, we have

$$\Delta\theta(F^0, F^1) = \underbrace{\left[\theta\left(F^0 + \Delta F_C^{(0)} + \Delta F_M^{(1)}\right) - \theta\left(F^0 + \Delta F_C^{(0)}\right)\right]}_{\Delta\theta_M^{(c)}} + \underbrace{\left[\theta\left(F^0 + \Delta F_C^{(0)}\right) - \theta(F^0)\right]}_{\Delta\theta_C^{(m)}} \tag{10}$$

and in the "marginal first", $(c, m) = (0,1)$,

$$\Delta\theta(F^0, F^1) = \underbrace{\left[\theta\left(F^0 + \Delta F_C^{(1)} + \Delta F_M^{(0)}\right) - \theta\left(F^0 + \Delta F_M^{(0)}\right)\right]}_{\Delta\theta_C^{(m)}} + \underbrace{\left[\theta\left(F^0 + \Delta F_M^{(0)}\right) - \theta(F^0)\right]}_{\Delta\theta_M^{(c)}} \tag{11}$$

The terms measure the contributions of copula and marginal distributions to changes in the summary index functionals.

2.3 Step 2: Detailed decompositions by income source

The 2-terms aggregate decomposition provides a general picture of the factors underlying the change in the distribution of household disposable incomes. It however begs the question of what specific income source drives these aggregate terms, is it earnings, or capital incomes or taxes and transfers (or their association with other sources)? We answer this by a further decomposition of each term, in a *(2k-1)*-terms detailed decomposition.

Decomposition of the marginal distributions term

The contribution of changes in the distribution of separate income sources to the aggregate marginal distributions component is easy to assess by varying marginal distributions one at a time. Recall that all k sources are ordered and labelled $1, \ldots, k$. The impact of source j can be assessed by

$$\Delta F_{M,j}^{(c)}(y) = \int_{-\infty}^{y} \int_{-\infty}^{y-y_1} \ldots \int_{-\infty}^{y-y_1-\ldots-y_{k-1}} dC_{1,\ldots,k}^{(c)}\left(F_1^1(y_1), \ldots, F_j^1(y_1), \ldots, F_k^0(y_k)\right) -$$

$$\int_{-\infty}^{y} \int_{-\infty}^{y-y_1} \ldots \int_{-\infty}^{y-y_1-\ldots-y_{k-1}} dC_{1,\ldots,k}^{(c)}(F_1^1(y_1), \ldots, F_j^0(y_1), \ldots, F_k^0(y_k)) \tag{12}$$

where the copula is held at reference value *c*, the marginal distributions of all sources $1 \leq j' < j$ are held at reference value 1 and the marginal distributions of all sources $j < j' \leq k$ are held at reference value 0. Applying Equation (12) to all sources defines a sequential decomposition of the change in marginal effect:

$$\Delta F_M^{(c)}(y) = \sum_{j=1}^{k} \Delta F_{M,j}^{(c)}(y). \tag{13}$$

Each term of the sum reflects the effect of the variation of a separate income component. By the sequential nature of the decomposition, the measured contribution of source *j* is contingent on the ordering of sources. The ordering of sources is a modelling decision independent on the decision about *c* and *m*. Unless a specific labelling sequence out of the possible *k!* sequences can be advocated, sensitivity of the measured contribution of source *j* over alternative sequences should be assessed. Alternatively all 2^{k-1} possible distinct values for the contribution of source *j* across the *k!* permutations can be combined in the Shapley value to obtain a unique contribution estimate (Shorrocks, 2013)

Decomposition of the copula term

The contribution of individual income sources to the impact of the copula is more complicated to define and assess. To decompose the copula term in sub-components attributable to changes in the rank dependence between separate income sources, we need to shift the rank correlation between these income sources from year 0 to 1 while keeping other factors unchanged. We start from the following representations of the distribution of total income obtained by *pooling* sources:

$$F^t(y) = \int_{-\infty}^{y} \int_{-\infty}^{y-y_1} \dots \int_{-\infty}^{y-y_1-\dots-y_{k-1}} dC_{1,\dots,k}^t \left(F_1^t(y_1), \dots, F_k^t(y_k)\right)$$

$$= \int_{-\infty}^{y} \int_{-\infty}^{y-y_1} \dots \int_{-\infty}^{y-y_1-\dots-y_{k-1}} dC_{1+2,\dots,k}^t \left(F_{1+2}^t(y_1+y_2), \dots, F_k^t(y_k)\right) \tag{14}$$

where the second equality is obtained by summing (pooling) sources 1 and 2 and the new terms F_{1+2}^t and $C_{1+2,\dots,k}^t$ are, respectively, the marginal distribution of the sum of sources 1 and 2, and the (k-1)-variate copula of the joint distribution of all sources after summing sources 1 and 2 (see Fan and Patton, 2014 on 'hierarchical copula' representations). Again, invoking Sklar's theorem, we have F_{1+2}^t

$$F_{1+2}^{t}(y) = \int_{-\infty}^{y}\int_{-\infty}^{y-y_1} dC_{1,2}^{t}\left(F_1^t(y_1), F_2^t(y_2)\right) \tag{15}$$

where $C_{1,2}^{t}$ is the bivariate copula of the joint distribution of sources 1 and 2. The distribution of total household income can thus be re-expressed in terms of two copulas $C_{1,2}^{t}$ and $C_{(1+2),\ldots,\mathrm{k}}^{t}$ and marginal univariate distributions.

To isolate the contribution of changes in the association between sources 1 and 2 to the overall "copula term", we form a counterfactual distribution by combining $C_{1,2}^{1}$ and $C_{1+2,\ldots,\mathrm{k}}^{0}$ and assessing how much the household income distribution changes from its base period (holding all marginal distributions to their reference period *m*):

$$\Delta F_{C,1+2}^{(m)}(y) =$$

$$\int_{-\infty}^{y}\int_{-\infty}^{y-y_1}\ldots\int_{-\infty}^{y-y_1-\cdots-y_{k-1}} dC_{y_{1+2},\ldots,y_k}^{0}\left(\int_{-\infty}^{y_1+y_2}\int_{-\infty}^{y_2} dC_{1,2}^{1}\left(F_1^{(m)}(y_1), F_2^{(m)}(y_2)\right), \ldots, F_k^{(m)}(y_k)\right) -$$

$$\int_{-\infty}^{y}\int_{-\infty}^{y-y_1}\ldots\int_{-\infty}^{y-y_1-\cdots-y_{k-1}} dC_{y_{1+2},\ldots,y_k}^{0}\left(\int_{-\infty}^{y_1+y_2}\int_{-\infty}^{y_2} dC_{1,2}^{0}\left(F_1^{(m)}(y_1), F_2^{(m)}(y_2)\right), \ldots, F_k^{(m)}(y_k)\right). \tag{16}$$

The only varying component in (16) is the bivariate copula for sources 1 and 2 (the dependence between the pair and all other sources are held fixed).

Working towards a full (k-1)-terms decomposition, this development is iterated by pooling source 3 to 1 and 2 (provided here $k>3$) and using the fact that

$$F^{t}(y) = \int_{-\infty}^{y}\int_{-\infty}^{y-y_1}\ldots\int_{-\infty}^{y-y_1-\cdots-y_{k-1}} dC_{1,\ldots,k}^{t}\left(F_1^t(y_1), \ldots, F_k^t(y_k)\right)$$

$$= \int_{-\infty}^{y}\int_{-\infty}^{y-y_1}\ldots\int_{-\infty}^{y-y_1-\cdots-y_{k-1}} dC_{1+2+3,\ldots,k}^{t}\left(F_{1+2+3}^{t}(y_1+y_2+y_3), \ldots, F_k^t(y_k)\right) \tag{17}$$

where F_{1+2+3}^{t} and $C_{1+2+3,\ldots,k}^{t}$ are the marginal distribution of the sum of sources 1, 2 and 3, and the (k-2)-variate copula of the joint distribution of all sources after summing sources 1, 2 and 3. The contribution of the change in the association between sources 1, 2 and 3 is given by

$$\Delta F_{C,(1+2)+3}^{(m)}(y) =$$

$$\int_{-\infty}^{y}\int_{-\infty}^{y-y_1}\dots\int_{-\infty}^{y-y_1-\dots-y_{k-1}} dC_{1+2+3,\dots,k}^{0}\left(\int_{-\infty}^{y_1+y_2+y_3}\int_{-\infty}^{y_2+y_3}\int_{-\infty}^{y_3} dC_{1,2,3}^{1}\left(F_1^{(m)}(y_1),F_2^{(m)}(y_2),F_3^{(m)}(y_3)\right),\dots,F_k^{(m)}(y_k)\right) -$$

$$\int_{-\infty}^{y}\int_{-\infty}^{y-y_1}\dots\int_{-\infty}^{y-y_1-\dots-y_{k-1}} dC_{1+2,\dots,\mathrm{k}}^{0}\left(\int_{-\infty}^{y_1+y_2}\int_{-\infty}^{y_2} dC_{1,2}^{1}\left(F_1^{(m)}(y_1),F_2^{(m)}(y_2)\right),\dots,F_k^{(m)}(y_k)\right). \quad (18)$$

Repeating this procedure until all sources are pooled and the copula becomes univariate $C_{1+2+3+\dots+k}^{t}\left(F_{1+2+3+\dots+\mathrm{k}}^{t}(y)\right) = F_{1+2+3+\dots+\mathrm{k}}^{t}(y)$ leads to the (k-1)-terms decomposition of the overall copula effect. Each term in the sequence reflects the contribution to the overall distributional change of the change in the association between a given income component and the (cumulative total of) previous sources in the sequence, holding its association with all other sources constant.

Again this decomposition is contingent on the labelling and ordering of sources and one can examine alternative permutations or estimate Shapley value averages.

2.4 *Estimation*

Having defined the various terms of the decomposition, we can turn to their estimation from household-level data. All counterfactual distributions used in defining the decompositions are easily estimated following Burtless' (1999) simulation strategy. The procedure is non-parametric, does not require any direct estimation of the copula functions and can also be applied to index functionals directly; see e.g. Daly & Valetta (2006) or Larrimore (2014) for more recent applications.

Determine first the fractional rank of household *i*'s income from source *j* in year *t* as per the rescaled empirical CDF (see, e.g., Fan & Patton, 2014),

$$r_{ij}^{t} = \hat{F}_j^{t}(y_{ij}^{t}) = \frac{1}{N+1}\sum_{h=1}^{N} I\{y_{hj}^{t} \leq y_{ij}^{t}\} \quad (19)$$

and note that

$$y_{ij}^{t} = \hat{Q}_j^{t}(r_{ij}^{t}) \quad (20)$$

where $\hat{Q}_{\mathrm{j}}^{t}$ is the generalized inverse (i.e. quantile function) of $\hat{F}_j^{t}$. The joint distribution of r_{ij}^{t} over all sources is the copula.

The aggregate decomposition requires simulation of the counterfactual distribution of total household incomes at times 0 and 1 holding the copula at period *c*. These distributions (and all index functionals of interest) are obtained using standard estimators from simulated household incomes where household *i*'s income at time *t* is simulated by holding ranks at period *c* and applying the period *t* quantile function:

$$\tilde{y}_i^{t,(c)} = \sum_{j=1}^{k} \quad \hat{Q}_j^t(r_{ij}^{(c)}) \quad . \tag{21}$$

Similarly, simulation of incomes holding marginal distributions constant is given by

$$\tilde{y}_i^{(m),t} = \sum_{j=1}^{k} \quad \hat{Q}_j^{(m)}(r_{ij}^t). \tag{22}$$

All terms of the decomposition can then be estimated by $\theta\left(\left\{\tilde{y}_i^{0,(c)}\right\}_{i=1}^N\right)$, $\theta\left(\left\{\tilde{y}_i^{1,(c)}\right\}_{i=1}^N\right)$, $\theta\left(\left\{\tilde{y}_i^{(m),0}\right\}_{i=1}^N\right)$ and $\theta\left(\left\{\tilde{y}_i^{(m),1}\right\}_{i=1}^N\right)$ where θ denotes a generic index functional estimator (or the empirical CDF estimator for the decomposition of the CDF).

For the detailed decomposition of the marginal distributions term, household incomes are simulated by holding fractional ranks to period *c* and the quantile functions of the first *j-1* sources to 1 and the others to 0

$$\tilde{y}_i^{t,j,(c)} = \sum_{j'=1}^{j-1} \quad \hat{Q}_{j'}^1(r_{ij}^{(c)}) \quad + \sum_{j'=j}^{k} \quad \hat{Q}_{j'}^0\left(r_{ij'}^{(c)}\right) \quad . \tag{23}$$

The counterfactual distributions needed in Equation (12) are again derived from the simulated vectors $\left\{\tilde{y}_i^{t,j,(c)}\right\}_{i=1}^N$ obtained for all sources.

Finally, the detailed decomposition of the copula term uses household counterfactual incomes defined as

$$\tilde{y}_i^{(m),t,j} = \hat{Q}_{1+\cdots+j}^1(r_{i,1+\cdots+j}^{(m)}) \quad + \sum_{j'=j+1}^{k} \quad \hat{Q}_{j'}^0(r_{ij'}^{(m)}) \tag{24}$$

where $\hat{Q}_{1+\cdots+j}^t$ is the period *t* quantile function of pooled income sources 1 to *j* and $r_{i,1+\cdots+j}^{(m)}$ is the corresponding fractional rank of household *i* in period *m* in the pooled income distribution.

Income ties and the uniqueness of copulas

Application of Sklar's theorem in Equation (3) determines a unique copula only when the marginal distributions are continuous; there is then a one-to-one relationship between the copula C and the multivariate distribution G. When at least one of the marginal distributions has discrete components (for example, several individuals have the same value of capital income), the copula is not unique and different copula functions are consistent with Equation (3) (e.g., Nelsen, 2006; Genest & Neslehova, 2007). Household income sources are mainly continuous except for a probability mass at zero when households do not receive a particular source of income; e.g. pension incomes or targeted benefits. This therefore implies that our decomposition approach defines a *family* of possible decomposition results. Identification of a unique decomposition requires additional assumptions. Building upon Rothe (2012, 2015), it is possible to make the copula unique by defining a set of k rank allocator functions which assign a unique "latent rank" to tied ranks and which are equal to observed ranks on continuous segments of the variables, preserving a monotonic relationship between the latent and observed ranks. We can build the rank allocator function by adding a small contamination ε_{ij}^t to all incomes, $y_{ij}^t + \varepsilon_{ij}^t$, where ε_{ij}^t is small enough that $y_{ij}^t + \varepsilon_{ij}^t < y_{i'j}^t$ for all $y_{i'j}^t > y_{ij}^t$ and $\varepsilon_{i'j}^t \neq \varepsilon_{ij}^t$ for all $i \neq i'$. The latent ranks are then given by the empirical CDF of the contaminated income values.

Three specific rank allocator functions are relevant. The first, which we follow in the main part of the empirical analysis, is to apply random assignment of ranks for tied observations, e.g., using random contamination terms ε_{ij}^t ('jittering'). This assumes away any correlation between the latent ranks of tied observations and other income sources (or total income). The other two rank allocators rank tied observations in one income source according to their positions in other income sources. In the first case, the rank allocator assigns latent ranks in increasing order of the sum of incomes from all other sources, namely, $y_i^t > y_{i'}^t \Rightarrow \varepsilon_{ij}^t > \varepsilon_{i'j}^t$ for any households i and i' with $y_{ij}^t = y_{i'j}^t$. This rank allocator effectively selects the copula that maximizes the correlation between source j latent ranks and total income ranks. In the second case, the rank allocator assigns unique latent ranks in decreasing order of the sum of incomes from all other sources, namely $y_i^t > y_{i'}^t \Rightarrow \varepsilon_{ij}^t < \varepsilon_{i'j}^t$ for any *i* and *i'* with $y_{ij}^t = y_{i'j}^t$. This selects a copula that minimizes the correlation between source *j* latent ranks and total income ranks.

3. The distribution of household disposable income in Luxembourg 2004-2013

We applied our decomposition to a study of the change in the household income distribution in Luxembourg between 2004 and 2013. Data are from the *Socio-Economic Panel "Liewen zu Lëtzebuerg"* (PSELL3), an annual representative survey on income and living conditions of individuals and private households. Total net household income is partitioned into seven components:

$$\text{Total net household income} = E_h + E_s + E_o + P + CI + PT - ITC \qquad (25)$$

where E_h is gross earnings of household head, E_s gross earnings of spouse, E_o gross earnings of other household members, *P* pensions, *CI* capital income, *PT* public transfers, and *ITC* income taxes and social security contributions. The gross earnings components include salary income and income from self-employment. Pension refers to old-age and survivor pensions and capital income to income from rent/land, interests and dividends from capital. In couple-headed households, we define the man as the head of the household, given the higher employment rate of men than women (Berger *et al.*, 2014), while in single-headed households the head of the household can be either a man or a woman. All income components are expressed as single-adult equivalent values using the modified OECD equivalence scale and deflated to 2005 prices.[5] The estimation sample comprises 8,994 individuals for 2004 and 9,963 individuals for 2013.

According to these data, household income inequality increased in Luxembourg between 2004 and 2013. A growth incidence curve (Figure 1) reveals a relatively simple pattern: whereas incomes at the bottom of the distribution have decreased, those at the top increased. Remarkably, the size of the income growth at the top mirrored the decline at the bottom. Mean income increased faster than median

[5] This scale gives the value 1 to the first adult, 0.5 to each additional adult and 0.3 to each child below 14.

income, the Gini coefficient and the P90/P10 ratio have risen significantly (Table 1). Relative poverty and richness rates—measuring the share of individuals above low income and high income thresholds—also rose (see Table 1 footnotes for detailed definitions).

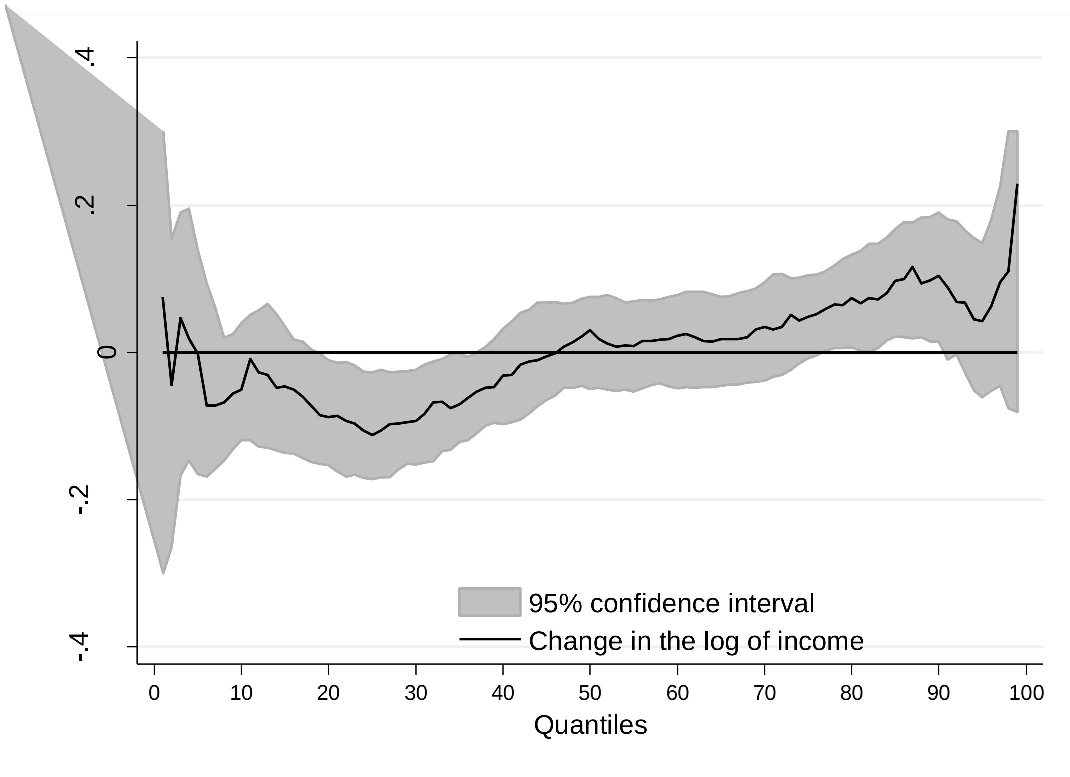

Figure 1. Growth incidence curve: The change in the distribution of household disposable income in Luxembourg between 2004 and 2013

Source: Authors' calculations based on the PSELL III, weighted data (cross-section weights).

Note: The Figure depicts the differences in the base-2 logarithms of income values between 2013 and 2004, at 99 equally spaced percentiles of the income distribution. The pointwise confidence intervals are derived using 500 bootstrap replications. For presentation purposes, we truncated the low bound confidence interval at the 1st percentile of the income distribution and the upper bound confidence interval at the 98-99th percentiles of the distribution.

Table 1. Changes in the distributional summary measures in Luxembourg between 2004 and 2013

Indexes	2004	2013	Change between 2013 and 2004	
			Estimate	Standard error
Mean income	31345.53	32586.49	+1241	(675.39)
Median income	27680.81	28221.04	+ 540	(655.93)
P90/P10	3.274	3.642	+0.368	(0.140)
P90/P50	1.801	1.902	+0.101	(0.056)
P50/P10	1.818	1.914	+0.096	(0.058)

Gini	0.266	0.303	+0.037	(0.010)
Poverty rate (%)	13.54	15.72	+2.18	(1.36)
Richness rate (%)	6.92	8.18	+1.26	(0.94)

Source: Authors' calculations based on the PSELL III, weighted data (cross-section weights).

Note: All measures are obtained for household equivalized disposable income. The relative poverty line is defined as 60% of the median household disposable income. The richness threshold is twice the median household disposable income in a given year. Standard errors are derived using 500 bootstrap replications.

So what has been driving these trends? A plot of the changes in the distribution of the seven components of income by percentiles (Figure 2) reveals no simple story.[6] Earnings of household heads tended to decline except for the highest 5 percent. On the other hand earnings of spouses, and other household members to a smaller extent, increased. Household incomes were also depressed by a decline in capital income and an increase in taxes, but pensions and other public transfers increased. The magnitudes of the changes and the share of households affected by them (measured by the length of the horizontal segments at zero) vary greatly across sources.

To complete the picture, Table 2 describes the pairwise rank and Pearson correlations between sources and their evolution between 2004 and 2013. The overall structure in both years is unsurprising, e.g., taxes are positively correlated with incomes (except public transfers) as a consequence of their progressivity, public transfers are negatively associated with market income sources given their role as social insurance, capital incomes are positively associated with pensions, reflecting the accumulation of assets by older population. Earnings of spouses are positively correlated—an indication of assortative mating—but both of spouses earnings are negatively correlated with earnings from other household members. The main changes between 2004 and 2013 were an increase in the correlation between capital and earnings, between earnings and public transfers (which became less negatively correlated), and a decline in the correlation between capital and public transfers.

[6] Table A.1 in Appendix A also summarizes changes in the average values of income sources.

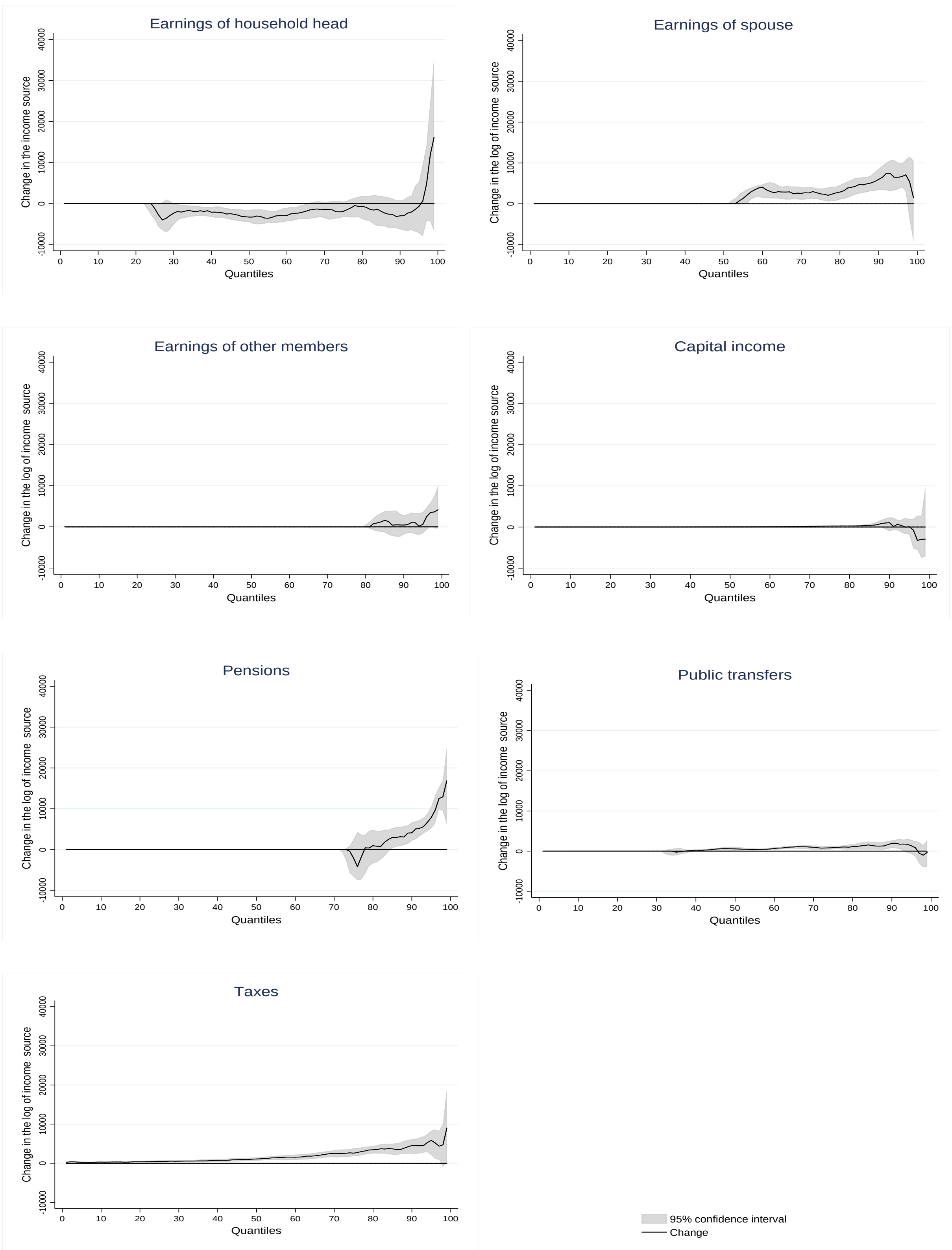

Figure 2. Changes in the marginal distributions of various income sources in Luxembourg between 2004 and 2013

Source: Authors' calculations based on the PSELL III, weighted data (cross-section weights).

Note: The Figure depicts the differences in the base-2 logarithms of income values between 2013 and 2004, at 99 equally spaced percentiles of the income distribution. The confidence intervals are derived using 500 bootstrap replications.

Table 2. Changes in the rank correlations of various income sources in Luxembourg between 2004 and 2013

Panel A: Pairwise correlations between income components in 2004

	Eh	Es	Eo	P	CI	PT	ITC	
Rank Eh	1.00	0.22***	-0.12***	-0.38***	0.03	-0.18***	0.70***	Eh
Rank Es	0.21***	1.00	-0.09***	-0.22***	-0.00	-0.05*	0.42***	Es
Rank Eo	-0.07***	-0.01	1.00	0.03	-0.02	-0.08***	0.06***	Eo
Rank P	-0.47***	-0.25	0.02	1.00	0.18***	-0.21***	-0.01	P
Rank CI	0.04*	0.01	0.01	0.06***	1.00	-0.04*	0.26***	CI
Rank PT	-0.03*	0.03	-0.06***	-0.25***	-0.07***	1.00	-0.10***	PT
Rank ITC	0.67***	0.29***	0.06***	-0.17***	0.16***	-0.22***	1.00	ITC
	Rank Eh	Rank Es	Rank Eo	Rank P	Rank CI	Rank PT	Rank ITC	

Panel B: Pairwise correlations between income components in 2013

	Eh	Es	Eo	P	CI	PT	ITC	
Rank Eh	1.00	0.17***	-0.09***	-0.24***	0.10***	-0.14***	0.86***	Eh
Rank Es	0.29***	1.00	-0.12***	-0.22***	+0.02	-0.09***	0.30***	Es
Rank Eo	-0.12***	-0.09***	1.00	+0.02	-0.01	-0.07***	0.02	Eo
Rank P	-0.46***	-0.27***	0.04**	1.00	0.14***	-0.24***	0.02	P
Rank CI	0.17***	0.08***	-0.05**	0.16***	1.00	-0.04**	0.23***	CI
Rank PT	-0.08***	0.07***	-0.02	-0.31***	-0.25***	1.00	-0.11***	PT
Rank ITC	0.58***	0.32***	0.01	-0.05**	0.44***	-0.31***	1.00	ITC
	Rank Eh	Rank Es	Rank Eo	Rank P	Rank CI	Rank PT	Rank ITC	

Panel C: Difference in the pairwise correlations between income components between 2013 and 2004

	Eh	Es	Eo	P	CI	PT	ITC	
Rank Eh	1.00	-0.05	+0.03	+0.14*	+0.07	+0.04	+0.16	Eh
Rank Es	+0.08**	1.00	-0.03	-0.00	+0.02	-0.04	-0.12	Es
Rank Eo	-0.05	-0.08*	1.00	-0.01	+0.01	+0.01	-0.04	Eo
Rank P	+0.01	-0.02	+0.02	1.00	-0.04	-0.03	+0.03	P
Rank CI	+0.13***	+0.07*	-0.06	+0.10**	1.00	-0.00	-0.03	CI
Rank PT	-0.05	+0.04	+0.04	-0.06*	-0.18***	1.00	-0.01	PT
Rank ITC	-0.09***	+0.03	-0.05	+0.12***	+0.28***	-0.09***	1.00	ITC
	Rank Eh	Rank Es	Rank Eo	Rank P	Rank CI	Rank PT	Rank ITC	

Source: Authors' calculations based on the PSELL III, weighted estimates.

Note: E_h is gross earnings of household head, E_s - gross earnings of spouse, E_o - gross earnings of other household members, *P* - pensions, *CI* - capital income, *PT* - public transfers, and *ITC* - income taxes and social security contributions. All income components are adjusted for the household size using the OECD-modified equivalized scale and are presented in the prices of 2013. The numbers above the diagonal represent Pearson correlation coefficients, the numbers under the diagonal stand for Spearman (rank) correlation coefficients. The differences between the periods are tested for significance using 500 bootstrap replications; * means significant at 0.05 level, ** means significant at 0.01 level, and *** means significant at 0.001 level.

4 Decomposition analysis

The contrasted evolution of earnings of heads and spouses, the larger sizes of taxes and transfers (and of their progressivity, as gauged by their correlation with market incomes), and the evolution of capital incomes emerge as the potential drivers of the increase in inequality. But the relative importance of these factors is difficult to assess from examining the marginal distributions or the correlation structure alone. Our decomposition sheds light on this.

4.1 Aggregate decomposition

Figure 3 shows aggregate decomposition results, namely how the change in the rank dependence between sources (Panel A) and their marginal distributions (Panel B) has reflected on the shift in the distribution of household disposable income in Luxembourg. We set $\{m,c\}$ to $\{0,1\}$: the change in the copula is assessed at 2004 marginal distributions and the change in the marginal distributions is assessed at 2013 dependence. The separate effects (shown as dashed lines in Figure 3) add up to the total change (shown as solid line). We use the 'independence' rank allocator to assign unique latent ranks to tied income sources (Section 4.3 provides assessment of the sensitivity of our results to alternative choices).

The decline in incomes at the bottom of the distribution (between the 10^{th} to 50^{th} percentiles) is mainly driven by the change in the copula. The relative decline in the bottom half of the distribution reflects an increased tendency of households to accumulate low incomes in multiple sources. Had the dependence structure of different income sources remained the same as in 2004, the decline in incomes at the bottom part of the household income distribution would not have taken place, other things being equal. By contrast, the shifts in the marginal distributions of income sources reflected predominantly on the upper half of the distribution. Panel B in Figure 3 shows that incomes increased from the 30^{th} percentile onwards and this increase explains alone the change in the household income distribution from the 50^{th} percentile (from which point the copula contribution disappears). Changes in marginal distributions also contributed to a fall in incomes in the bottom 10 percent.

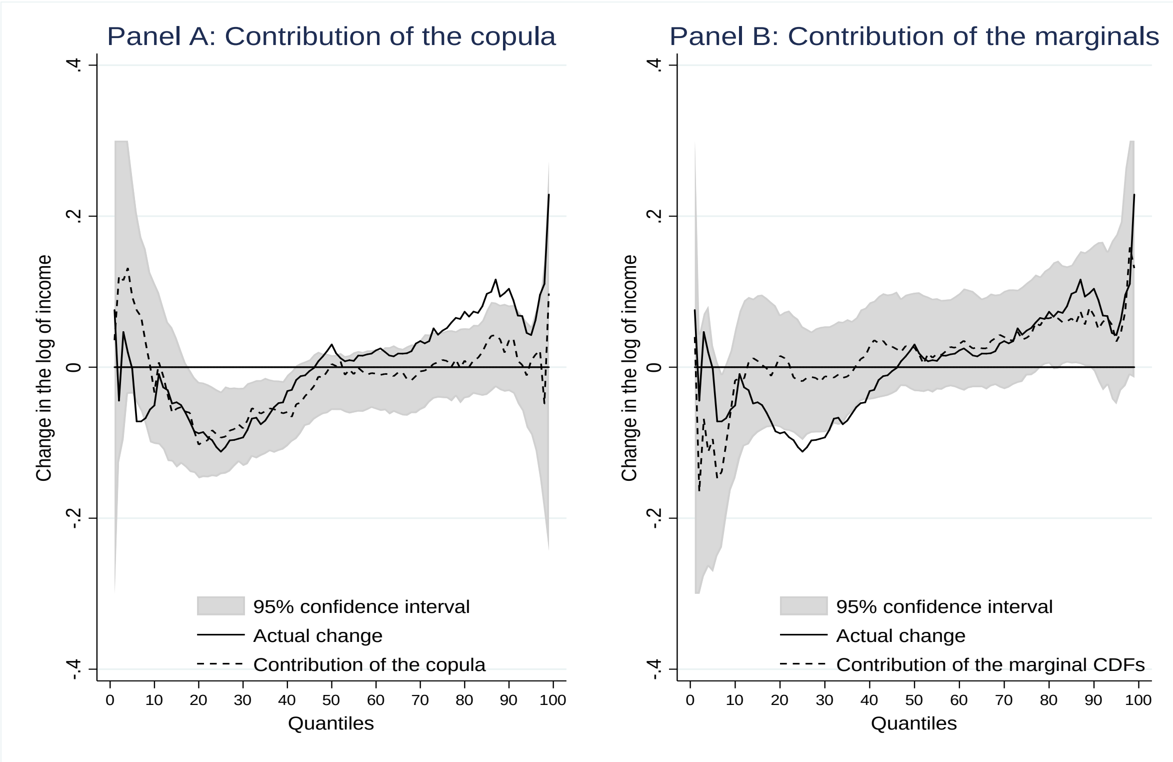

Figure 3. Aggregate decomposition of the change in the household disposable income in Luxembourg between 2004 and 2013

Source: Authors' calculations based on the PSELL III, weighted data (cross-section weights).

Note: The Figure depicts the differences in the base-2 logarithms of real income values at 99 equally spaced percentiles of the income distribution. The line 'actual change' captures the difference in actual income values between 2013 and 2004. The contributions of the decomposition components (changes in the copula and marginal CDFs of income sources) are calculated according to Equation (8) in the text. The confidence intervals are derived using 500 bootstrap replications.

Both components were dis-equalizing forces: the copula by reducing incomes in (most of) the bottom half of the household income distribution and the marginals by mostly increasing incomes in its upper half. This is reflected in Table 3 which provides decomposition results for summary measures. Both changes in the marginal distributions of income sources and their dependence structure are associated with the increase in all inequality and poverty measures. About 2/3 of the increases in the Gini coefficient and the P90/P10 ratio is ascribed to the shifts in the marginal distributions of income sources, and 1/3 is ascribed to the copula. By contrast, the change in the copula accounts for a substantial proportion of the increase in the relative poverty rate. According to the estimates, had the rank association between income sources remained the same as in 2004, we would have observed 63.4% smaller increase in the relative poverty rate in 2013.

Table 3. Aggregate decomposition of the change in the distribution of household equivalized disposable income in Luxembourg between 2004 and 2013

Distributional summary measure	Overall change	Change due to the copula		Change due to the marginal CDFs of income sources	
		Estimate (SE)	In %	Estimate (SE)	In %
Gini	+0.029	+0.008	+26.15	+0.021	+73.85
	(0.011)	(0.008)		(0.013)	
P90/P10	+0.362	+0.103	+28.51	+0.259	+71.49
	(0.139)	(0.173)		(0.156)	
P90/P50	+0.099	+0.045	+45.99	+0.053	+54.01
	(0.054)	(0.052)		(0.048)	
P50/P10	+0.095	+0.009	+9.11	+0.087	+90.89
	(0.057)	(0.086)		(0.073)	
Poverty rate, %	+2.029	+1.286	+63.41	+0.742	+36.59
	(1.30)	(1.31)		(1.07)	
Richness rate, %	+0.861	+0.058	+6.82	+0.802	+93.18
	(0.89)	(0.78)		(0.81)	

Source: Authors' calculations based on the PSELL III, weighted data (cross-section weights).

Note: Overall changes in the distributional summary measures between 2004 and 2013 are expressed in absolute terms and calculated using predicted values of incomes (for more details, see Section 2.4). The standard errors in the parentheses are based on 500 bootstrap replications.

4.2 *Detailed decomposition results*

The detailed decomposition results look inside the aggregate components and point to the evolution of specific sources or correlation across sources and its contribution to the change in household income distribution. The detailed decompositions require a sequencing of sources. Moving from market to disposable income sources seems the most natural option: we start with market incomes (earnings-related first)—earnings of head, of spouse, of other members, pensions, capital incomes—and then move to disposable incomes by adding public transfers and finally deducting income taxes paid.

Let us examine the contribution of marginal distributions first. Figure 4 shows detailed results for the contributions of the marginal CDFs to the shift in the distribution of household disposable income. Changes in the marginal distributions of all income sources (with the exception of earnings of other household members and capital income) had a significant contribution to the shift in the distribution of household disposable income. The change in the distribution of earnings of household heads—which declined on average by 8 percent—resulted in a decline of incomes along the entire distribution of household income (except of its top 10 percent), with an especially profound decline documented at the very bottom of the distribution, a potentially strongly inequality-increasing force. The change in the marginal CDF of earnings of spouses—which, by contrast, increased on average by 18 percent— is associated with a relatively constant, inequality-neutral, increase of all percentiles of the total household income distribution. This evidence is in line with findings for other rich countries (see, e.g., Burtless, 1999, Daly & Valletta, 2006, and Larrimore, 2014) which show the increased share of spouse's (typically female) earnings in total household income.

Figure 4 also reveals a substantial contribution of pensions to income growth along the entire distribution of household incomes, with larger increases observed higher up the distribution. By contrast to market income sources, changes in the marginal distributions of public transfers and taxes changed in a way to equalize incomes. The change in the marginal distribution of public transfers was associated with the increase in incomes of all households, but especially those at the bottom. Changes in the marginal distribution of taxes had an opposite effect.

Table 4 completes the graphical evidence by quantifying the contributions of changes in the marginal distribution of each income source to distributional summary measures. As expected from the graphical inspection, the change in the marginal distribution of household heads' earnings was the major driver of the increase in all inequality and poverty measures. The change in the marginal distribution of pensions was the second most important contributor to the increase. These evolutions were partially offset by the shifts in the distributions of public transfers and taxes. Around 50% of the increase in the Gini coefficient induced by the shift in the marginal distribution of earnings of household heads was offset by the changes in the marginal distributions of taxes and transfers.

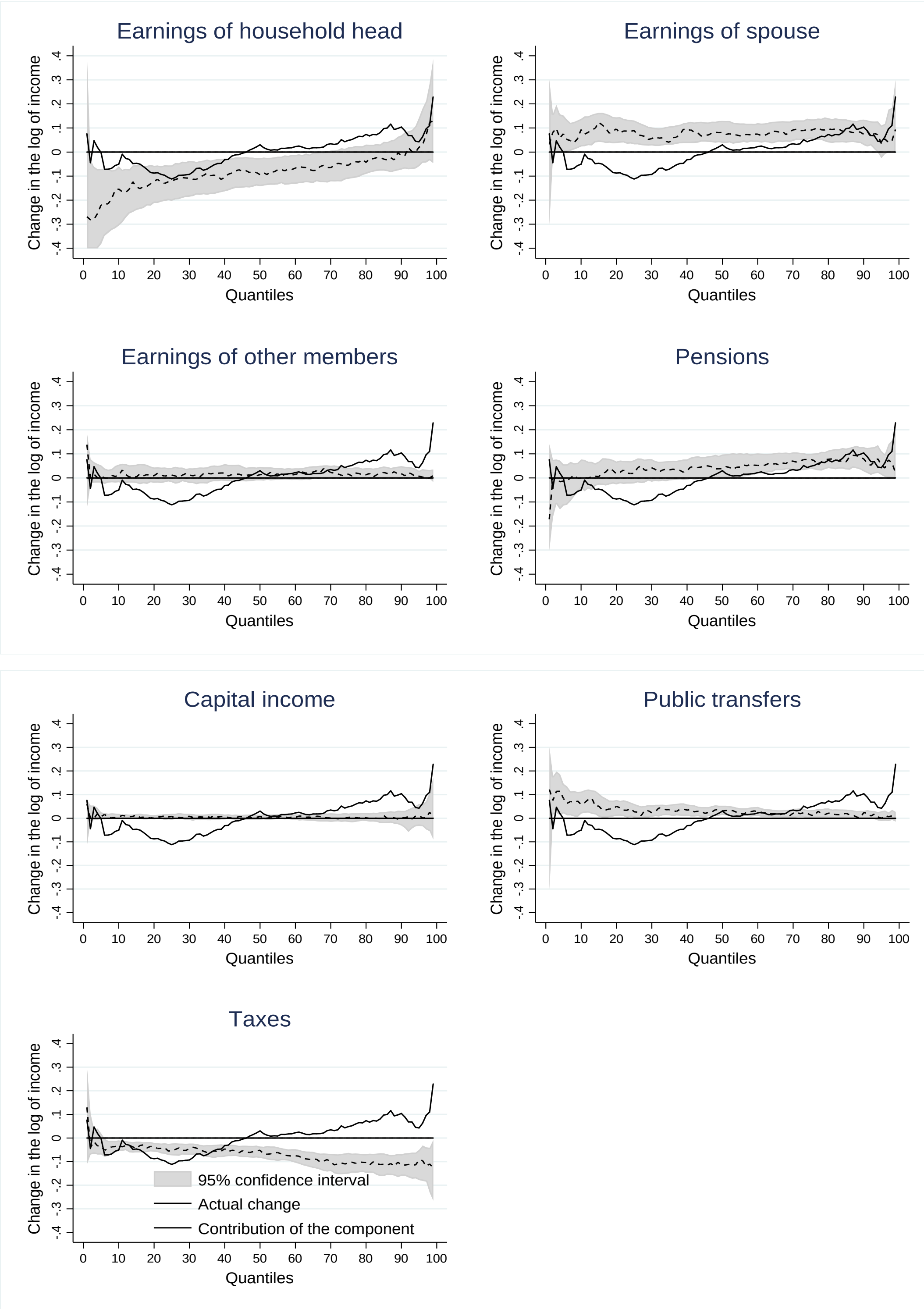

Figure 4. Detailed decomposition of the marginal distributions component

Source: Authors' calculations based on the PSELL III, weighted data (cross-section weights).

Note: The line 'actual change' captures the actual difference in the base-2 logarithm of real income values between 2012 and 2003 at 99 equally spaced percentiles of the income distribution. The contributions of the decomposition components are identified in a sequence following the algorithm described in Section 5 and represent changes in the distribution of household disposable income associated with the respective factors. The confidence intervals are derived using 500 bootstrap replications.

Table 4. Detailed decomposition of the change in the distribution of household disposable income in Luxembourg between 2004 and 2013

Decomposition component	Contribution to the change in the following measures: Gini	P90/P10	P90/P50	P50/P10	Poverty rate	Richness
Overall change	+0.029	+0.362	+0.099	+0.096	+2.03	+0.86
	(0.011)	(0.139)	(0.054)	(0.057)	(1.30)	(0.90)
Eh with Es	+0.010	+0.252	+0.063	+0.079	+2.44	+0.66
	(0.004)	(0.095)	(0.036)	(0.047)	(1.01)	(0.55)
Eo with Eh, Es	-0.003	-0.050	+0.026	-0.056	-0.99	+0.36
	(0.002)	(0.078)	(0.020)	(0.047)	(1.00)	(0.30)
P with Eh, Es, Eo	+0.003	+0.117	-0.006	+0.078	+1.33	+0.10
	(0.003)	(0.096)	(0.030)	(0.059)	(1.28)	(0.51)
CI with Eh, Es, Eo, P	+0.007	+0.079	+0.044	+0.003	+0.24	+0.55
	(0.001)	(0.042)	(0.022)	(0.016)	(0.42)	(0.35)
PT with Eh, Es, Eo, P, CI	-0.007	-0.115	-0.008	-0.065	-1.59	+0.02
	(0.002)	(0.073)	(0.022)	(0.043)	(1.06)	(0.36)
ITC with Eh, Es, Eo, P, CI, PT	-0.002	-0.180	-0.073	-0.030	-0.14	-1.63
	(0.006)	(0.140)	(0.044)	(0.078)	(1.16)	(0.74)
Total due to the copula	+0.008	+0.103	+0.046	+0.009	+1.29	+0.06
	(0.008)	(0.173)	(0.052)	(0.086)	(1.31)	(0.78)
Eh	+0.030	+0.353	+0.100	+0.093	+1.58	+1.80
	(0.011)	(0.146)	(0.036)	(0.065)	(0.96)	(6.94)
Es	-0.001	-0.019	-0.017	+0.006	-0.16	-0.07
	(0.003)	(0.084)	(0.032)	(0.041)	(0.008)	(0.004)
Eo	-0.000	+0.019	+0.011	-0.001	+0.05	-0.13
	(0.001)	(0.043)	(0.019)	(0.019)	(0.52)	(0.24)
P	+0.007	+0.169	+0.049	+0.047	+0.73	+0.49
	(0.003)	(0.068)	(0.033)	(0.024)	(0.68)	(0.40)
CI	-0.000	-0.006	-0.007	+0.004	-0.00	+0.03
	(0.002)	(0.030)	(0.014)	(0.008)	(0.21)	(0.26)
PT	-0.006	-0.079	-0.008	-0.037	-1.03	-0.46
	(0.001)	(0.047)	(0.012)	(0.025)	(0.60)	(0.002)
ITC	-0.009	-0.178	-0.075	-0.025	-0.43	-0.86
	(0.005)	(0.054)	(0.025)	(0.018)	(0.42)	(0.47)
Total due to marginal CDFs	+0.021	+0.259	+0.053	+0.087	+0.74	+0.80
	(0.013)	(0.156)	(0.048)	(0.073)	(1.07)	(0.81)

Source: Authors' calculations based on the PSELL III, weighted data (cross-section weights).

Note: E_h is gross earnings of household head, E_s - gross earnings of spouse, E_o - gross earnings of other household members, *P* - pensions, *CI* - capital income, *PT* - public transfers, and *ITC* - income taxes and social security contributions. All income components are adjusted for the household size using the OECD-modified equivalized scale. The standard errors are derived using 500 bootstrap replications.

Let us turn to the detailed decomposition of the aggregate copula term. Figure 5 reveals that it is the change in the rank dependence in market incomes—between earnings of household heads and spouses as well as in the rank dependence of capital income with all earnings-related sources—that has been associated with most of the relative decline in incomes at the bottom of the distribution. In other words, individuals who score low in one market income source became more likely over time to score low also in other income sources, which contributed to the decline in their relative incomes. By contrast, increases in the rank dependence of public transfers and taxes with other income sources—reflecting both higher effective targeting of public transfers and greater tax progressivity—were predominantly beneficial to those at the bottom of the income distribution. These increases, however, did not offset the inequality-increasing impacts of the association of market income sources.

Table 4 quantifies these contributions on distributional summary measures. Changes in the rank dependence of earnings of household heads with the earnings of spouses, as well as changes in the rank dependence of capital income with other market income sources, have contributed significantly to the increase in inequality and poverty indexes. The increased rank correlation between earnings of heads and spouses is associated with the largest increase in all inequality and poverty measures. Around 1/3 of the total increase in the Gini coefficient and 2/3 of the total increase in all percentile ratios can be assigned to the changed dependence between earnings of household heads and spouses. The contribution is stronger for the poverty rate – had the copula between earnings of household heads and spouses remained unchanged, the poverty rate would have been lower in 2013 compared to 2004. The change in the rank correlation of capital income with other market income sources is associated with a statistically significant decline in incomes at the bottom and in the middle of the household income distribution. As a consequence the distribution has become more unequal in 2013 as compared to 2004, shifting the Gini coefficient and percentile ratios up. Almost 30% of the increase in the Gini coefficient can be attributed to the change in the rank dependence between capital income and other earnings-related income sources. This increase has been partially offset by the changes in the rank correlation of these income sources with transfers. Similarly, the change in the rank dependence of taxes with other income sources contributed to partly offsetting dis-equalization of the distribution of household disposable income.

4.3 Sensitivity to modelling decisions

We performed two sets of sensitivity checks to assess the robustness of our results with respect to (1) the choice of rank allocator, that is, the treatment of tied observations in income components, and (2) the choice of $\{c,m\}$, that is, the order of the decomposition between the two main components.

The treatment of tied observations -- Alternative rank allocator functions

We replicated analysis using the minimum and maximum dependence rank allocators presented in Section 2.4. The results are presented in Tables B.1 and B.2 in Appendix B.

In general, we find that at the level of the detailed decomposition the estimates based on two alternative rank allocators are very close to the estimates from the main results. The change in the rank correlation of earnings of household heads with earnings of spouses has the largest dis-equalizing contribution to the shift in the distribution of total household income within the copula block. The change in the dependence structure of capital income with other market income sources also significantly contributed to the increase in inequality, regardless of how one treats the tied observations in the sample. The change in the rank correlations of public transfers with all market income sources and the change in the rank correlation of taxes with all other income components are persistently associated with a decrease in inequality and poverty indexes.

Regarding the contributions of the changes in marginal CDFs of the income sources, the results from the two alternative rank allocators are very close to our baseline results. Change in the marginal CDFs of earnings of household heads and pensions had a dis-equalizing influence on the distribution of total household income, while the change in the marginal CDFs of public transfers and taxes is associated with a decline in all measures.

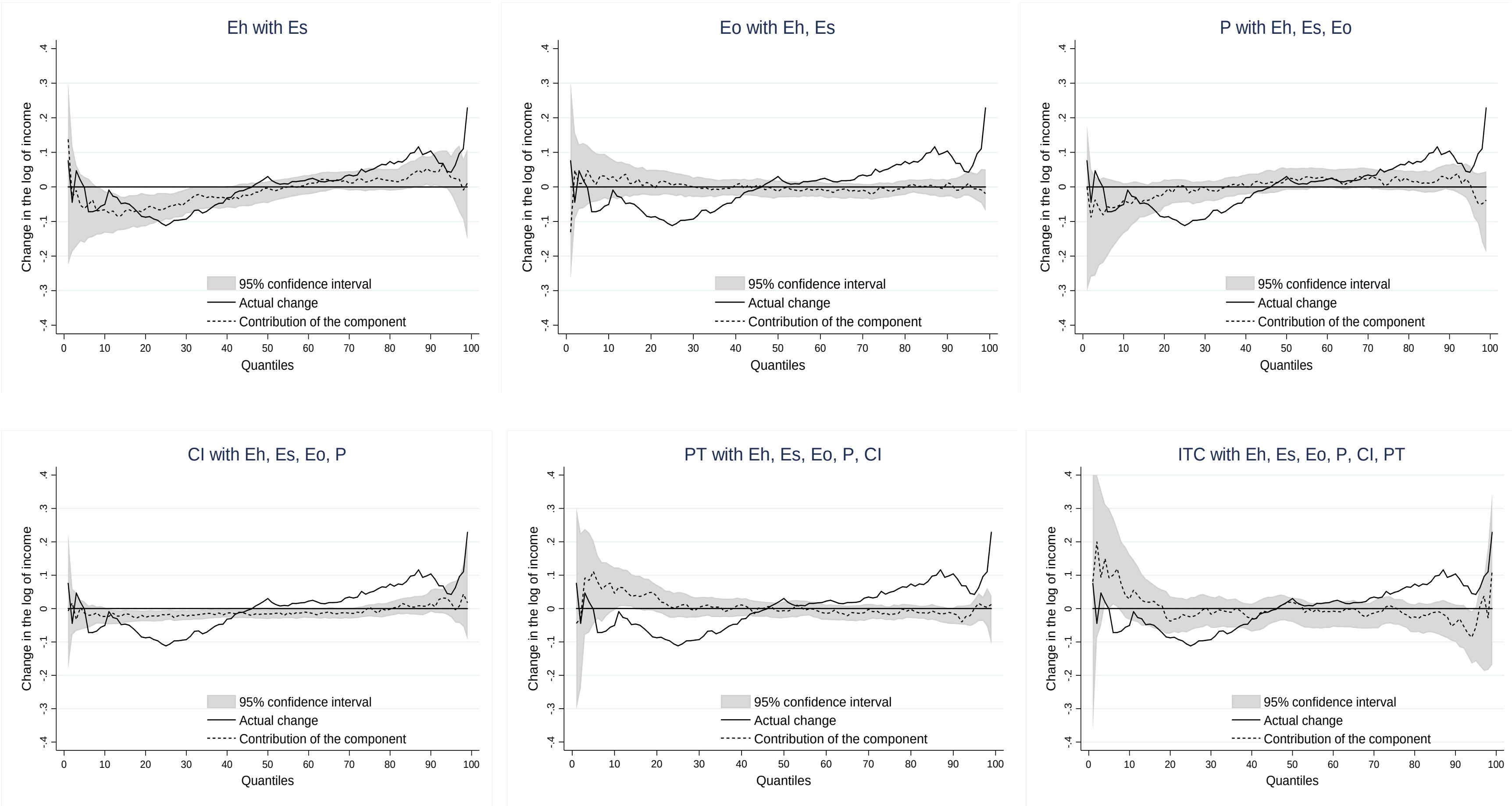

Figure 5. Detailed sequential decomposition of the copula component

Source: Authors' calculations based on the PSELL III, weighted data (cross-section weights).

Note: The line 'actual change' captures the actual difference in the base-2 logarithm of real income values between 2012 and 2003 at 99 equally spaced percentiles of the income distribution. The contributions of the decomposition components are identified in a sequence following the algorithm described in Section 5 and represent changes in the distribution of household disposable income associated with the respective factors. The confidence intervals are derived using 500 bootstrap replications.

A reverse order decomposition

Second, we implemented a reverse order decomposition by identifying first the contribution of the marginal CDFs of income sources before the contribution of the copula-related components, that is set $\{m,c\}$ to $\{1,0\}$. The results are again similar to our main findings in Section 4.1 and 4.2. In the aggregate decomposition, the size of the copula contribution to the shifts in the distributional summary measures, however, is larger when the contribution of the marginal CDFs of income sources is derived before the contribution of the copula (see Table C1 in on-line Appendix C). The estimates of the copula contributions gained precision and became significant for the Gini coefficient and the P90/P10 and P90/P50 percentile ratios. By contrast, most of the contributions of the changes in the marginal CDFs of income sources became smaller in size and largely insignificant.

Results are also very similar to the baseline estimates in the detailed decomposition. The estimates of the copula contributions become larger in size and also gain significance for a number of distributional summary measures. The nature of the contributions, however, remains the same. The main difference is that the contribution of the change in the rank correlation of pensions with other market income sources becomes significant.

5 Conclusion

This paper proposes a simulation-based decomposition of changes in the distribution of household incomes into two main components: (i) changes in the marginal distributions of different sources of income—labour incomes, capital incomes, transfers, taxes—and (ii) changes in the correlation between these sources. Each component is further disaggregated to distinguish the impacts of changes in each of the separate sources of income. While a number of studies have applied similar principles in counterfactual simulation exercises – e.g., Burtless (1999), Fournier (2001) or Larrimore (2014) – formalizing the decomposition in terms of the joint distribution of income sources, marginal and copula functions clarifies the rationale and interpretation of the simulations and resulting terms, something that has not been done in the literature for disaggregate decompositions to date.

The decomposition is illustrated on the change in the distribution of total household disposable income in Luxembourg between 2004 and 2013. The distribution of household disposable income had become more unequal in Luxembourg between those two years, exhibiting an increase in inequality and poverty measures. The decomposition reveals that the shift in the distribution has been induced by both changes in the marginal distributions of income sources and their dependence. While changes in the marginal distributions of income sources can be held predominantly responsible for the growth of incomes in the upper part of the income distribution, the change in association between income sources accounts for the decline in incomes in its lower tail, so both factors contribute to increasing inequality but through different channels. The shifts in the marginal distributions of income sources explain most of the increase in inequality measures and the richness rate whereas the change in the copula is associated with a substantial portion of the increase in the relative poverty rate. Predominantly well-off households have benefited from the growth of incomes coming from various sources over time. The households at the bottom of the distribution not only have not enjoyed this growth but also became more likely to rank low in multiple income sources. Increased progressivity of taxes and transfers only partially offset this increased correlation of earnings-related sources.

Appendix A

Table A1. Changes in the mean values of various income sources in Luxembourg between 2004 and 2013

Income source	2004	2013	Change between 2013 and 2004	
			Estimate	Standard error
Earnings of household head	14.35	14.27	-0.08	(0.05)
Earnings of spouse	13.24	13.42	+0.18	(0.08)
Earnings of other household members	12.92	13.00	+0.08	(0.15)
Capital income	9.86	7.55	-2.31	(0.20)
Pensions	14.01	14.15	+0.14	(0.10)
Public transfers	11.81	12.05	+0.24	(0.05)
Taxes	12.18	12.50	+0.32	(0.05)

Source Authors' calculations based on the PSELL III, weighted data (cross-section weights).
Note: All income values are in base 2 logarithms. The standard errors are derived using 500 bootstrap replications.

Appendix B

Table B.1. Decomposition of the change in the distribution of household disposable income in Luxembourg between 2004 and 2013 (assumption of maximum inequality)

Decomposition component	Contribution to the change in the following measures					
	Gini	P90/P10	P90/P50	P50/P10	Poverty	Richness
Overall change	+0.029	+0.369	+0.101	+0.098	+2.14	+0.85
	(0.011)	(0.139)	(0.054)	(0.057)	(1.30)	(0.90)
Eh with Es	+0.012	+0.281	+0.085	+0.072	+2.23	+1.00
	(0.004)	(0.103)	(0.039)	(0.047)	(1.01)	(0.56)
Eo with Eh, Es	-0.003	-0.047	+0.021	-0.050	-0.62	+0.17
	(0.002)	(0.078)	(0.022)	(0.048)	(0.99)	(0.32)
P with Eh, Es, Eo	+0.003	+0.127	-0.005	+0.084	+1.16	+0.22
	(0.004)	(0.098)	(0.032)	(0.059)	(1.28)	(0.50)
CI with Eh, Es, Eo, P	+0.008	+0.073	+0.050	-0.005	+0.37	+0.54
	(0.002)	(0.041)	(0.022)	(0.016)	(0.43)	(0.33)
PT with Eh, Es, Eo, P, CI	-0.007	-0.140	-0.018	-0.070	-1.68	+0.06
	(0.002)	(0.074)	(0.023)	(0.044)	(1.07)	(0.37)
ITC with Eh, Es, Eo, P, CI, PT	-0.011	-0.254	-0.100	-0.043	-0.20	-2.19
	(0.007)	(0.160)	(0.047)	(0.080)	(1.19)	(0.80)
Total due to the copula	+0.002	+0.040	+0.033	-0.012	+1.26	-0.18
	(0.008)	(0.182)	(0.053)	(0.087)	(1.30)	(0.80)
Eh	+0.031	+0.337	+0.083	+0.100	+1.58	+1.60
	(0.011)	(0.155)	(0.035)	(0.069)	(0.96)	(0.63)
Es	+0.003	+0.069	+0.022	+0.016	-0.02	+0.33
	(0.003)	(0.082)	(0.028)	(0.041)	(0.84)	(0.47)
Eo	-0.000	+0.004	+0.002	+0.000	-0.02	-0.15
	(0.001)	(0.042)	(0.019)	(0.019)	(0.52)	(0.23)
P	+0.007	+0.161	+0.043	+0.049	+0.81	+0.48
	(0.003)	(0.069)	(0.033)	(0.025)	(0.69)	(0.41)
CI	+0.001	+0.011	-0.001	+0.007	-0.01	+0.09
	(0.002)	(0.030)	(0.014)	(0.007)	(0.20)	(0.26)
PT	-0.006	-0.079	-0.007	-0.039	-1.03	-0.47
	(0.002)	(0.048)	(0.012)	(0.026)	(0.006)	(0.002)
ITC	-0.009	-0.174	-0.074	-0.023	-0.43	-0.85
	(0.005)	(0.054)	(0.025)	(0.018)	(0.43)	(0.47)
Total due to marginal CDFs	+0.027	+0.329	+0.068	+0.110	+0.88	+1.03
	(0.013)	(0.163)	(0.048)	(0.075)	(0.99)	(0.80)

Note: E_h is gross earnings of household head, E_s - gross earnings of spouse, E_o - gross earnings of other household members, *P* - pensions, *CI* - capital income, *PT* - public transfers, and *ITC* - income taxes and social security contributions. All income components are adjusted for the household size using the OECD-modified equivalized scale. The standard errors are derived using 500 bootstrap replications.

Table B.2. Decomposition of the change in the distribution of household disposable income in Luxembourg between 2004 and 2013 (assumption of minimum inequality)

Decomposition component	Contribution to the change in the following measures					
	Gini	P90/P10	P90/P50	P50/P10	Poverty	Richness
Overall change	+0.029	+0.363	+0.095	+0.100	+2.15	+0.86
	(0.011)	(0.139)	(0.055)	(0.057)	(1.30)	(0.90)
Eh with Es	+0.008	+0.260	+0.076	+0.070	+2.24	+0.85
	(0.004)	(0.103)	(0.037)	(0.051)	(0.012)	(0.006)
Eo with Eh, Es	-0.003	-0.076	+0.010	-0.055	-0.63	+0.21
	(0.002)	(0.078)	(0.020)	(0.048)	(1.01)	(0.30)
P with Eh, Es, Eo	+0.003	+0.155	+0.004	+0.089	+1.05	+0.07
	(0.003)	(0.094)	(0.032)	(0.056)	(1.31)	(0.50)
CI with Eh, Es, Eo, P	+0.007	+0.070	+0.052	-0.008	+0.26	+0.66
	(0.002)	(0.043)	(0.022)	(0.016)	(0.43)	(0.34)
PT with Eh, Es, Eo, P, CI	-0.007	-0.134	-0.014	-0.070	-1.49	-0.00
	(0.002)	(0.075)	(0.023)	(0.044)	(1.10)	(0.37)
ITC with Eh, Es, Eo, P, CI, PT	+0.006	-0.073	-0.085	+0.039	-0.14	-1.68
	(0.006)	(0.129)	(0.045)	(0.069)	(1.40)	(0.76)
Total due to the copula	+0.014	+0.202	+0.042	-0.065	+1.29	+0.11
	(0.009)	(0.172)	(0.053)	(0.086)	(1.42)	(0.78)
Eh	+0.030	+0.270	+0.108	+0.039	+1.82	+1.65
	(0.011)	(0.138)	(0.036)	(0.062)	(1.08)	(0.68)
Es	-0.006	-0.011	-0.021	+0.016	-0.27	+0.01
	(0.004)	(0.100)	(0.033)	(0.047)	(1.00)	(0.49)
Eo	-0.001	+0.020	+0.010	+0.001	+0.21	-0.11
	(0.002)	(0.041)	(0.019)	(0.019)	(0.51)	(0.24)
P	+0.008	+0.160	+0.044	+0.048	+0.58	+0.45
	(0.003)	(0.067)	(0.033)	(0.025)	(0.69)	(0.40)
CI	-0.001	-0.022	-0.005	-0.007	-0.02	+0.07
	(0.002)	(0.029)	(0.015)	(0.007)	(0.23)	(0.26)
PT	-0.006	-0.078	-0.007	-0.037	-1.07	-0.46
	(0.002)	(0.048)	(0.013)	(0.026)	(0.62)	(0.23)
ITC	-0.009	-0.178	-0.074	-0.025	-0.39	-0.86
	(0.005)	(0.054)	(0.025)	(0.018)	(0.42)	(0.48)
Total due to marginal CDFs	+0.015	+0.161	+0.053	+0.035	+0.86	+0.75
	(0.014)	(0.162)	(0.048)	(0.077)	(1.15)	(0.80)

Note: E_h is gross earnings of household head, E_s - gross earnings of spouse, E_o - gross earnings of other household members, *P* - pensions, *CI* - capital income, *PT* - public transfers, and *ITC* - income taxes and social security contributions. All income components are adjusted for the household size using the OECD-modified equivalized scale. The standard errors are derived using 500 bootstrap replications.

Appendix C

Table C.1. A reverse order decomposition of the change in the distribution of household disposable income in Luxembourg between 2004 and 2013

Decomposition component	Contribution to the change in the following measures					
	Gini	P90/P10	P90/P50	P50/P10	Poverty	Richness
Overall change	+0.029	+0.362	+0.099	+0.096	+2.03	+0.86
	(0.011)	(0.139)	(0.054)	(0.057)	(1.30)	(0.90)
Eh with Es	+0.01	+0.303	+0.115	+0.050	+1.77	+2.12
	(0.004)	(0.111)	(0.042)	(0.050)	(1.03)	(0.77)
Eo with Eh, Es	-0.003	-0.096	+0.012	-0.066	-1.14	-0.01
	(0.002)	(0.076)	(0.021)	(0.044)	(0.89)	(0.31)
P with Eh, Es, Eo	+0.002	+0.219	-0.023	+0.145	+2.69	-0.49
	(0.003)	(0.103)	(0.022)	(0.059)	(1.06)	(0.35)
CI with Eh, Es, Eo, P	+0.007	+0.082	+0.039	+0.008	+0.29	+0.88
	(0.002)	(0.047)	(0.023)	(0.015)	(0.34)	(0.39)
PT with Eh, Es, Eo, P, CI	-0.007	-0.213	-0.028	-0.092	-2.22	-0.42
	(0.002)	(0.079)	(0.019)	(0.045)	(0.91)	(0.33)
ITC with Eh, Es, Eo, P, CI, PT	+0.003	-0.022	-0.006	-0.006	-0.41	-0.70
	(0.002)	(0.061)	(0.028)	(0.027)	(0.50)	(0.49)
Total due to the copula	+0.013	+0.273	+0.109	+0.039	+0.98	+1.38
	(0.005)	(0.136)	(0.051)	(0.064)	(1.35)	(0.91)
Eh	+0.029	+0.401	+0.101	+0.116	+2.51	+1.30
	(0.011)	(0.125)	(0.037)	(0.050)	(1.08)	(0.64)
Es	-0.002	-0.059	+0.006	-0.039	-1.13	+0.31
	(0.004)	(0.081)	(0.027)	(0.036)	(0.84)	(0.52)
Eo	-0.001	-0.022	-0.006	-0.006	+0.05	+0.05
	(0.002)	(0.054)	(0.018)	(0.024)	(0.51)	(0.33)
P	+0.005	+0.136	-0.017	+0.092	+2.01	-0.47
	(0.003)	(0.070)	(0.032)	(0.024)	(0.67)	(0.47)
CI	+0.000	-0.009	-0.003	-0.002	+0.000	-0.19
	(0.002)	(0.031)	(0.016)	(0.008)	(0.19)	(0.27)
PT	-0.007	-0.188	-0.034	-0.069	-1.78	-0.24
	(0.002)	(0.056)	(0.012)	(0.027)	(0.67)	(0.23)
ITC	-0.008	-0.170	-0.057	-0.035	-0.61	-1.28
	(0.005)	(0.059)	(0.027)	(0.018)	(0.49)	(0.51)
Total due to marginal CDFs	+0.016	+0.089	-0.010	+0.057	+1.05	-0.52
	(0.013)	(0.146)	(0.046)	(0.062)	(1.23)	(0.85)

Note: E_h is gross earnings of household head, E_s - gross earnings of spouse, E_o - gross earnings of other household members, *P* - pensions, *CI* - capital income, *PT* - public transfers, and *ITC* - income taxes and social security contributions. All income components are adjusted for the household size using the OECD-modified equivalized scale. The standard errors are derived using 500 bootstrap replications.

Appendix D

Table D.1. Decomposition of the change in the distribution of household disposable income in Luxembourg (taking 2013 as the base year)

Decomposition component	Contribution to the change in the following measures					
	Gini	P90/P10	P90/P50	P50/P10	Poverty	Richness
Overall change	-0.029	-0.360	-0.095	-0.099	-2.03	-0.85
	(0.012)	(0.139)	(0.055)	(0.057)	(1.39)	(0.96)
Eh with Es	-0.011	-0.244	-0.103	-0.020	-1.45	-1.72
	(0.005)	(0.171)	(0.057)	(0.079)	(1.05)	(0.83)
Eo with Eh, Es	+0.003	+0.068	-0.036	+0.069	-0.17	-0.27
	(0.003)	(0.126)	(0.029)	(0.061)	(0.90)	(0.42)
P with Eh, Es, Eo	-0.001	-0.151	+0.057	-0.129	-1.50	+0.78
	(0.004)	(0.158)	(0.039)	(0.078)	(1.24)	(0.63)
CI with Eh, Es, Eo, P	-0.009	-0.123	-0.015	-0.147	-0.88	-0.14
	(0.005)	(0.072)	(0.026)	(0.028)	(0.51)	(0.46)
PT with Eh, Es, Eo, P, CI	+0.007	+0.316	+0.025	+0.130	+1.46	+0.19
	(0.002)	(0.150)	(0.031)	(0.079)	(1.13)	(0.48)
ITC with Eh, Es, Eo, P, CI, PT	-0.002	-0.141	-0.041	-0.040	+1.53	-0.29
	(0.006)	(0.165)	(0.054)	(0.079)	(1.28)	(0.91)
Total due to the copula	-0.013	-0.275	-0.113	-0.037	-1.01	-1.45
	(0.006)	(0.138)	(0.053)	(0.067)	(1.45)	(0.93)
Eh	-0.031	-0.407	-0.060	-0.150	-2.82	-1.21
	(0.013)	(0.158)	(0.052)	(0.056)	(1.09)	(0.83)
Es	+0.003	+0.060	-0.045	+0.074	+1.21	-0.53
	(0.003)	(0.118)	(0.032)	(0.049)	(0.80)	(0.51)
Eo	+0.001	+0.032	+0.007	+0.010	+0.38	-0.07
	(0.002)	(0.058)	(0.023)	(0.022)	(0.53)	(0.36)
P	-0.003	-0.180	+0.020	-0.110	-2.26	+0.68
	(0.003)	(0.074)	(0.030)	(0.029)	(0.75)	(0.55)
CI	-0.000	+0.048	+0.009	+0.015	-0.11	+0.25
	(0.002)	(0.029)	(0.012)	(0.010)	(0.22)	(0.25)
PT	+0.007	+0.190	+0.032	+0.065	+1.71	+0.32
	(0.002)	(0.083)	(0.013)	(0.035)	(0.75)	(0.29)
ITC	+0.007	+0.172	+0.055	+0.034	+0.87	+1.16
	(0.003)	(0.054)	(0.024)	(0.017)	(0.47)	(0.45)
Total due to marginals	-0.016	-0.085	+0.018	-0.062	-1.02	+0.60
	(0.011)	(0.153)	(0.049)	(0.064)	(1.21)	(0.87)

Note: E_h is gross earnings of household head, E_s - gross earnings of spouse, E_o - gross earnings of other household members, *P* - pensions, *CI* - capital income, *PT* - public transfers, and *ITC* - income taxes and social security contributions. All income components are adjusted for the household size using the OECD-modified equivalized scale. The standard errors are derived using 500 bootstrap replications.

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