

---

**WORKING PAPERS**

Combining microsimulation and  
optimization to identify optimal  
universalistic tax-transfer rule

Ugo **COLOMBINO**<sup>1,2</sup>  
Nizamul **ISLAM**<sup>2</sup>

*LISER Working Papers are intended to make research findings available and stimulate comments and discussion. They have been approved for circulation but are to be considered preliminary. They have not been edited and have not been subject to any peer review.*

*The views expressed in this paper are those of the author(s) and do not necessarily reflect views of LISER. Errors and omissions are the sole responsibility of the author(s).*

# Combining microsimulation and optimization to identify optimal universalistic tax-transfer rule<sup>1</sup>

**Ugo Colombino**

University of Turin, Italy, and Luxembourg Institute of Socio economic Research (LISER), Luxembourg.

**Nizamul Islam**

Luxembourg Institute of Socio Economic Research (LISER), Luxembourg.

09 2021

## Abstract

As a response to a changing labour market scenario and to the concerns for increasing costs and bad incentives of traditional income support policies, the last decades have witnessed, in many countries, reforms introducing more sophisticated designs of means-testing, eligibility and tagging. In this paper, we consider an alternative direction of reform that points towards universality, unconditionality and simplicity. Our main research question is whether tax-transfer rules designed according to these alternative criteria might be superior to the current one and could therefore be proposed as a policy reform. We adopt a computational approach to the design of optimal tax-transfer rules, within a flexible class. The exercise is applied to France, Germany, Italy, Luxembourg, Spain and the United Kingdom. The results suggest some common features in all the countries. The optimal tax-transfer rules feature a universal unconditional basic income or, equivalently, a negative income tax with a guaranteed minimum income. The tax profiles are much flatter than the current ones. For most social welfare criteria, and most countries, the simulated tax-transfer rules are superior to the current ones. These results confirm that policy reforms inspired by the principle of Universal Basic Income and Flat Tax might have good chances to dominate the current tax-transfer rules.

**JEL:** H2, H3, C6

**Keywords:** Empirical Optimal taxation, Microsimulation, Microeconometrics, Social Welfare evaluation of Tax-Transfer rules, Computational economics.

---

<sup>1</sup> This research is part of the “Optimal Tax Rules in Europe: A Micro-simulation Approach (OTaRiE)” project supported by core funding for LISER from the Ministry of Higher Education and Research of Luxembourg. The authors would like to thank for useful comments and critiques the participants of the Workshop on Labour Income Taxation and the Labour Supply, LISER, Luxembourg, 20 January 2020. Ugo Colombino gratefully acknowledges financial and organizational support from LISER.

## 1. Introduction

During the last decades, a variety of social, economic and technological processes has been leading to a new labour market scenario: fewer available jobs, more intermittent careers; more mobility across jobs with different locations, contents and required skills; more heterogeneity of income sources. At the same time, it emerged a concern for increasing costs of traditional income support policies, due to both a higher demand for support and to welfare “traps” or dependence. The new scenario and its implications represented a challenge to the traditional welfare policies, which – as a response – tried to evolve toward a more sophisticated design of means-testing, eligibility and tagging, aimed at sustaining work incentives (e.g., through in-work benefits or tax credits) and at matching specific needs and attitudes of different sociodemographic groups with categorical policies.<sup>2</sup> In spite of some appreciable results on labour participation incentives, these policies have been criticised for introducing new inequalities, distortions and high monitoring costs.<sup>3</sup> The policy debate, however, considers also an alternative direction of reform that aims at over-coming (or complementing) means-testing and categorical policies, pointing towards universality, unconditionality and simplicity.<sup>4</sup> In this paper, we adopt this latter alternative view. We consider a simple though flexible class of TTRs, where net available household income is defined as 4-th degree polynomial in taxable income plus a constant term that is scaled according to the household’s size. The rule is universally applied to all the households. Within this class of TTRs, we look for the social-welfare-optimal member according to the criterion proposed by Kolm (1976), analysing the data of six European countries: France, Germany, Italy, Luxembourg, Spain and the United Kingdom. Our research question is whether the social-welfare-optimal members of the considered class are social-welfare superior to the current TTRs. We formulate the problem as an instance of empirical optimal taxation. However, we do not adopt the familiar analytical approach, not in the original version (e.g., Mirrlees 1971) nor in the “sufficient statistics” version (e.g. Saez 2001, 2002). We develop a computational approach that combines microeconomic modelling, microsimulation, numerical optimization and social welfare evaluation. We identify

---

<sup>2</sup> Analyses of these reforms are provided by, among others, Eissa and Liebman (1996), Shoemi and Blabk (2000), Blank (2002), Moffit (2003), Francesconi et al. (2009).

<sup>3</sup> See for example Bryan (2005), Handler and Babcock (2006), Standing (2011).

<sup>4</sup> See for example: Atkinson (1996), Colombino (2019), Colombino and Islam (2018), Colombino and Narazani (2013), Gentilini et al. (2020), Ghatak and Jaravel (2020), Ghatak and Maniquet (2019), Grimalda et al. (2020), Islam and Colombino (2018), Magnani and Piccoli (2020), Moene and Raj (2016), Standing (2012), Van Parijs and Vanderborght (2017).

optimal TTRs for different degrees of social inequality aversion and compare them to the current rules. With respect to the literature on empirical optimal taxation, this paper provides three main contributions that are relevant both from the methodological and the policy point-of-view.

First, it considers a flexible – even though parametric – class of TTRs. This is important both for guaranteeing the generality of our results and for allowing the analysis of issues that are crucial in tax-transfer reforms, such as the size of a guaranteed minimum income, the degree of progressivity and the trade-off between efficiency and equality. The heterogeneity accounted for in the data and in the microeconomic model in principle might allow us to consider TTRs based on some categorical articulation of tax rates and subsidies, which might be welfare-superior to our optimal polynomial TTRs. However, categorical and means-tested designs of the TTR bear administrative and political costs that instead are smaller or even non-existent in simple and universalistic designs. In view of policy reforms, our research motivation consists precisely of testing the performance of a very simple, transparent and universalistic TTR against the current (typically categorical and means-tested) TTR.

Second, most of the quantitative analysis of tax-transfer reforms in the last three decades are dedicated to mean-tested and categorical policies, which are also the prevailing policies in the considered countries. However, more recently, the policy debate has witnessed an increasing interest in unconditional and universal policies.<sup>5</sup> Our analysis permits an evaluation of the latter approach using a polynomial tax-transfer rules as compared to the prevailing current one.

Third, we identify some significant effects of “primitives” (i.e., basic characteristic of the economy) on the features of the optimal polynomial Tax Transfer Rules (TTRs). Despite the common features, the results show also large differences between the optimal polynomial TTRs in the different countries. They depend indeed on various characteristics of the population and the economic environment. An explanation of these differences requires to identify a general relationship between the basic (“primitive”) characteristics of the economy and the features of the optimal TTRs. Actually, this is the direct result of analytical optimal taxation. We can come close

---

<sup>5</sup> Besides the references of footnote 3, there is abundant evidence upon experiments in developing countries focussing on universal basic income and many pilots or projects also in developed countries: e.g. <https://www.visualcapitalist.com/map-basic-income-experiments-world/>.

to a similar result by identifying a “mapping” from the set of country-specific “primitives” to the set of country-specific optimal TTRs.

For most countries and most values of the inequality aversion parameter, the optimal polynomial rules provide a higher social welfare than the current ones. The results suggest some common features in all the countries. The optimal polynomial TTRs envisage a universal unconditional basic income with a (almost) flat tax or, depending on the interpretations, a (almost) flat negative income tax with guaranteed minimum. The profiles of the MTRs are pretty similar in different countries and definitely flatter than under the current TTRs. Section 2 presents the computational approach (a very brief survey of the analytical approaches is provided in Appendix A). Section 3 describes the microeconomic model and the procedure adopted to identify the optimal country-specific TTRs and a general rule for mapping from the “primitives” to some features of the optimal TTRs (Appendix B provides a more detailed description of the econometric model and reports the estimates for the couples and the singles in the six countries and the simulated elasticities of labour supply). Section 4 illustrates the results and Section 5 concludes.

## **2. The computational approach to empirical optimal taxation**

In the basic framework of optimal taxation theory, the Government chooses the taxes to be applied to households’ personal incomes with the aim of maximizing some social welfare criterion that accounts for both total welfare and its distribution among the households. While doing so, the Government takes into account a public budget constraint - i.e., the taxes must collect a given amount (to be used in public expenditures) – and an incentive constraint, i.e. household incomes and paid taxes are determined by household utility maximizing (labour supply) choices subject to budget constraints that are also defined by taxes. Using appropriate analytical optimization techniques one can obtain, under convenient assumptions, closed-form expressions for the optimal tax rate for a given level of taxable income. The analytical optimal taxation, pioneered by Mirrlees (1971), is a fundamental contribution since it sets the basic problem to be solved. Its empirical applications (e.g. Mirrlees 1971, Tuomala 1990, 2010, Saez 2001) can also indicate promising directions of policy reform. However, the relevance of those results and policy prescriptions critically depends on the flexibility and generality of the assumptions and on the ability to account

for the heterogeneity and realistic features of the economies.<sup>6</sup> In view of the above perspective, we will propose and apply a computational approach. A combination of micro-econometric modelling, behavioural microsimulation and numerical optimization provides an alternative or a complement with respect to the analytical approach.<sup>7</sup> Modern micro-econometric models of labour supply can be specified according to general and flexible assumptions. They can account for many realistic features such as heterogeneous preferences and opportunity sets, simultaneous decisions of couples, complicated budget constraints, multidimensional heterogeneity of both households and jobs, quantitative constraints, etc. It might not be feasible or practical to obtain analytical solutions for the optimal taxation problem in such economic environments. Yet those features are likely to be relevant and important for evaluating or designing reforms. The ability to adopt more general assumptions and to account for realistic features of the population and of the opportunities might provide more robust policy prescriptions. The implementation of the computational approach is articulated in four steps.

First, we estimate a microeconomic model of household labour supply. The model accounts for both singles and couples, wage employed, self-employed and non-participants, extensive and intensive labour supply responses, multidimensional sources of welfare, heterogeneous preferences and quantity constraints.

Second, for any member of a given class of TTRs, we can simulate the new household choices based on the estimated household preferences and compute the attained value of households' utility.

Third, we apply a maximization algorithm and iterate step two in order to identify the TTR that maximizes a Social Welfare function. The Social Welfare function takes as arguments (an appropriate transformation of) the previously computed household utility level.<sup>8</sup>

---

<sup>6</sup> Appendix A provides a brief overview of the analytical approach.

<sup>7</sup> The background of the computational approach is exemplified by a series of papers: Islam and Colombino (2018) identify optimal TTRs within the Negative Income Tax matched with a Flat Tax (NIT+FT) class in eight European countries. Aaberge and Colombino (2006, 2013) identify optimal taxes for Norway within the class of 9-parameter piece-wise linear TTRs. Aaberge and Colombino (2012) perform a similar exercise for Italy. Blundell and Shephard (2012) design an optimal TTR for lone mother in the UK. Closely related contributions are Fortin et al. (1993), Sefton and Van de Ven (2009), Creedy and Hérault (2012), Ericson and Flood (2012) and Colombino (2015).

<sup>8</sup> Of course one might adopt different evaluation criteria, such as the effects on employment, poverty etc. We adopt a social welfare criterion but we will also report and discuss results on other indices that might be policy-relevant.

Fourth, obtaining a general rule. The first three steps explained above will identify a specific optimal polynomial TTR for each country. Given the country-specific optimal TTRs and a set of country-specific “primitives” (i.e. basic characteristics of the economy) we can identify the mapping from the “primitives” to the optimal TTRs, i.e. a general rule analogous to the one identified by the analytical approach. As a matter of fact, the path of the computational approach is opposite to path of the analytical approach. The latter solves for a general rule and then can obtain country-specific rules by assigning country-specific values to the “primitives”. The former identifies country-specific rules and from those rules a general rule can be inferred. The general rule can be used for many purposes, e.g. providing indications for tax reforms in countries where reliable micro data are not available; making “out-of-sample” predictions in order to test the whole optimal taxation procedure; forecasting the need for fiscal reforms based on predictions about trends or future changes of the “primitives”.<sup>9</sup>

---

<sup>9</sup> In principle, the analytical approach is more general regarding the representation of the optimal TTR, which is non-parametric, while our approach adopts a parametric specification. However, the non-parametric representation comes at the price of restrictive assumptions upon other elements of the problem. Moreover, the non-parametric solutions produced by the analytical approach typically produce shapes of the optimal TTR that can be easily approximated by parametric polynomial functions (e.g. Heathcote and Tsujiyama 2015).



### 3. Model, Evaluation criterion and Optimization

*The microeconomic model.* A detailed description of the microeconomic model is given in Appendix B. In this Section we present the main elements. The household chooses one alternative from an opportunity set contains jobs that belong to different types indexed by  $j$ . Type are defined by hours of work ( $\mathbf{h}$ ) and sector of employment  $s$  (wage employment or self-employment). Non-market activities (“leisure”) are “jobs” that will be indexed as  $j = 0$ . There are  $M$  types, including  $j = 0$ . The utility level attained by household  $i$  when holding a job of type  $j$  given wage rate  $w_i$  and TTR  $\tau$  (a vector of parameters that define the tax-transfer rule) is written as follows,

$$U_i(j; w_i, \tau, \varepsilon_{ij}) = V_i(j; w_i, \tau) + \varepsilon_{ij} \quad (1)$$

where  $V_i(j; w_i, \tau)$  is the “systematic” part (a parametric function of observed variables: income, hours of work and socio-economic characteristics of the household) and  $\varepsilon_{ij}$  is a random variable that accounts for unobserved features of the match  $(i, j)$ .  $w_i$  is a scalar for singles and a vector for couples. Aaberge and Colombino (2013) presents a “Random Utility Random Opportunity (RURO)” approach where the probability that household  $i$  holds a job is -

$$P_i(j; w_i, \tau) = \frac{\exp\{V_i(j; w_i, \tau) + \delta_j D_j\}}{\sum_{x=0}^M \exp\{V_i(x; w_i, \tau) + \delta_x D_x\}} \quad (2)$$

where the  $D$  are alternative-specific dummies (see Appendix B for more details).

The estimates for couples (32 parameters), singles females (17 parameters) and single males (17 parameters) and labour supply elasticities in all the five countries are reported in Tables B1- B13 of Appendix B.

**The class of polynomial TTRs.** We look for optimal TTRs within the class of rules defined as a polynomial functions of total household taxable income  $y_i = \mathbf{w}_i \mathbf{h}_i + I_i - \text{SSC}_i$  where  $\text{SSC}_i$  denotes social security contributions. Net available income  $C_i$  is specified as follows:

$$C_i = \tau_0 \sqrt{N_i} + \tau_1 y_i + \tau_2 y_i^2 + \tau_3 y_i^3 + \tau_4 y_i^4 \quad (3)$$

where  $y_i$  (= total taxable household income) and  $N_i$  = household size. The choice of this simple specification is due to three main motivations. First, since we compare six different countries, our results are made more easily interpretable by abstracting from details and keeping the optimal TTRs as simple as possible. Second, even though the 4<sup>th</sup> degree polynomial specification is a parametric, it is flexible enough to be judged close to a non-parametric rule. Third, our specific research interest is investigating whether a very simple and universalistic TTR can outperform the (typically categorical and complex) current TTRs.

The corresponding TTR is:

$$T(y_i; \boldsymbol{\tau}) = y_i - \tau_0 \sqrt{N} - \tau_1 y_i - \tau_2 y_i^2 - \tau_3 y_i^3 - \tau_4 y_i^4 \quad (4)$$

The marginal tax rate and the average tax rate are respectively:

$$MT(y; \boldsymbol{\tau}) = \frac{\partial T(y; \boldsymbol{\tau})}{\partial y} = 1 - \tau_1 - 2\tau_2 y - 3\tau_3 y^2 - 4\tau_4 y^3 \quad (5)$$

$$AT(y; \boldsymbol{\tau}) = \frac{T(y; \boldsymbol{\tau})}{y} = 1 - \frac{\tau_0 \sqrt{N}}{y} - \tau_1 - \tau_2 y - \tau_3 y^2 - \tau_4 y^3$$

The rule is sufficiently flexible to represent many alternative versions of TTRs. Provided  $\tau_0 > 0$ , the rule can be interpreted as a negative income tax or a basic income matched with a generic tax rule.<sup>10</sup> In the former case  $\tau_0 \sqrt{N_i}$  is the universal guaranteed minimum income when  $y_i = 0$ , in the

---

<sup>10</sup> In all the optimal TTRs we obtain  $\tau_0 > 0$ . The equivalence of a universal basic income and a universal negative income tax with guaranteed minimum income can be easily seen in the flat tax case, although it carries over to non-flat taxes. Consider first the universal basic income. The household receives  $\tau_0 \sqrt{N_i}$  and pays taxes on its own taxable income  $y_i$  according to a constant marginal tax rate  $1 - \tau_1$ , so that its net disposable income will be

latter case it is a universal basic income. The case  $C_i = \tau_0 \sqrt{N_i} + \tau_1 y_i$  corresponds to a NIT+FT, i.e. the TTR class considered in Islam and Colombino (2018). The term  $\sqrt{N_i}$  rescales the guaranteed minimum income or the basic income according to the household size (square root rule). A pure flat tax rule is the special case  $C_i = \tau_1 y_i$ . Also rules with negative marginal taxes (such as In-work Benefits or Tax Credits) are accounted for, depending on the values of the parameters  $\tau$ .

When identifying the optimal TTR, the rule of expression (4) completely replaces the current TTR. A correct interpretation of the comparison of the optimal TTR to the current one must take into account important differences between the two. First, the rule of expression (4) applies to the sum of all household personal taxable incomes, whatever the source. The current rules might use different rules depending on the source and might apply to individual or household incomes. Second, the current income support mechanisms are typically a combination of (mostly) means-tested and categorical transfers. The rule of expression (4), instead, envisages a universal mechanism that can be interpreted as a guaranteed minimum income or as a basic income, provided  $\tau_0 > 0$ . In the evaluation of the relative performance of the optimal polynomial TTRs as compared to the current TTRs, we can only conclude that a certain TTR is better or worse (according to a given criterion) to another one. We cannot identify the specific contribution of, say, income support mechanisms, or the treatment of different income sources, to the relative performance of optimal TTRs as compared to current TTRs. However, in order to get a summary comparison of the current TTR and the optimized TTRs, we compute an approximation to the current TTR, which will be

---

$C_i = \tau_0 \sqrt{N_i} + \tau_1 y_i$ . With the negative income tax, it is useful to define the exemption level  $E_i = \frac{\tau_0 \sqrt{N_i}}{1 - \tau_1}$ . If taxable

income  $y_i$  is smaller than  $E_i$  the household receives a transfer  $(E_i - y_i)(1 - \tau_1)$  so that its net disposable income will

be  $C_i = y_i + (E_i - y_i)(1 - \tau_1) = y_i + \left( \frac{\tau_0 \sqrt{N_i}}{1 - \tau_1} - y_i \right) (1 - \tau_1) = \tau_0 \sqrt{N_i} + \tau_1 y_i$ . If taxable income is larger than  $E_i$  the

household only pays a tax  $(y_i - E_i)(1 - \tau_1)$  so that its net available income will be

$C_i = y_i - (y_i - E_i)(1 - \tau_1) = y_i - \left( y_i - \frac{\tau_0 \sqrt{N_i}}{1 - \tau_1} \right) (1 - \tau_1) = \tau_0 \sqrt{N_i} + \tau_1 y_i$ . Despite the budget-wise equivalence, when

it comes to policy implementation, universal basic income and universal negative income tax might differ in their implications for their administrative costs, the timing of transfers and the perception by the households. However, these aspects are beyond the scope of our analysis.

used in commenting the results. The approximation is the 4<sup>th</sup> degree polynomial that satisfies the public budget constraint and minimizes the sum of squared differences between the household observed disposable income and the household disposable income computed according to expression (3).

**Welfare evaluation.** We define the Comparable Money-metric Utility (CMU). This concept is based on the approach proposed by King (1983), where different preferences are due to different characteristics within a common parametric utility function. The characteristics account for a different productivity in obtaining utility from the opportunities available in the budget set. The utility levels attained by households with different preferences are made comparable by using a common “reference” household. The CMU of a given household  $i$  is the level of income that the “reference” household would need to attain the same utility level attained by household  $i$ . The procedure is analogous to using a reference price vector in order to compare utility levels attained under different price vectors. Empirical examples of this approach are provided by King (1983), Aaberge et al. (2004) and Islam and Colombino (2018). Our CMU transforms the household utility level into an inter-household comparable monetary measure that will enter as argument of the Social Welfare function. First, we calculate the expected maximum utility attained by household  $i$

under tax-transfer regime  $\tau$  (McFadden 1978):  $\ln\left(\sum_j \exp\{V_i(j; w_i, \tau)\}\right)$ . Analogously, we define

$\ln\left(\sum_j \exp\{V_R(j; w_R, \tau_R)\}\right)$  as the expected maximum utility attained by the “reference” household

$R$  under the “reference” tax-transfer regime  $\tau_R$ . The reference household is the couple household at the median value of the distribution of the expected maximum utility. The reference TTR  $\tau_R$  is a pure flat tax that satisfy the public budget constraint. The CMU of household  $i$  under tax regime  $\tau$ ,  $\mu_i(\tau)$ , is defined as the gross income that a reference household under a reference tax-transfer regime  $\tau_R$  would need in order to attain an expected maximum utility attained under tax-transfer rule to  $\tau$ . Although the choice of the reference household is essentially arbitrary, some choices make more sense than others. Our choice of the median household as the reference household can be justified in terms of representativeness or centrality of its preferences. Aaberge and Colombino

(2006, 2013) adopt a related, although not identical, procedure that consists of using a common utility function as argument of the social welfare function (Deaton and Muelbauer 1980).

In order to aggregate the household-specific welfare levels, we choose the Social Welfare index proposed by Kolm (1976), which can be defined as:

$$W = \bar{\mu} - \frac{1}{k} \ln \left[ \sum_i \frac{\exp\{-k(\mu_i - \bar{\mu})\}}{N} \right] \quad (6)$$

$W$  has limit  $\bar{\mu}$  as  $k \rightarrow 0$  and  $\min\{\mu_1, \dots, \mu_N\}$  as  $k \rightarrow \infty$ .

$\bar{\mu} = \frac{1}{N} \sum_i \mu_i$  is an index of efficiency

$$\frac{1}{k} \ln \left[ \sum_i \frac{\exp\{-k(\mu_i - \bar{\mu})\}}{N} \right] = \text{Kolm's Inequality Index} \quad (7)$$

$k$  = inequality aversion parameter<sup>11</sup>

$\mu_i$  = comparable money-metric utility of household  $i$  (defined in Section 3.3).

Therefore, Social Welfare = Efficiency – Inequality. Social Welfare and its components are monetary measures. The Inequality Index can be interpreted as the cost of inequality.<sup>12</sup>

**Identification of optimal TTRs.** The problem to be solved can be written as follows:

---

<sup>11</sup> In this paper we identify optimal TTRs for six value of  $k$ : 0.0, 0.05, 0.075, 0.10, 0.125, and 0.15. It can be shown (Islam and Colombino 2018) that the corresponding values of the popular Atkinson's parameter of inequality aversion are 0.000, 0.114, 0.180, 0.252, 0.333 and 0.424.

<sup>12</sup> Kolm's Inequality Index is an absolute index, meaning that it is invariant with respect to translations (i.e. adding a constant to every  $\mu_i$ ). Absolute indices are less popular than relative indices (e.g. Gini's or Atkinson's), although there is no strict logical or economic motivation for preferring one to the other. Atkinson and Brandolini (2010) provide a discussion of relative indexes, absolute indexes and intermediate cases. Depending on the specific application, there might be a motivation of computational convenience for choosing one or another type of index. Blundell and Shephard (2012) adopt a social welfare index which turns out to be very close to Kolm's index. Their main motivation for their index seems to be computational, since it handles negative numbers (random utility levels). Our motivation is analogous.

$$\begin{aligned}
& \max_{\boldsymbol{\tau}} W\left(\sum_i \mu_i(\boldsymbol{\tau})\right) \\
& \text{s.t.} \\
& \sum_i \sum_j P_{ij}(\boldsymbol{\tau}) T_{ij}(\boldsymbol{\tau}) \geq R
\end{aligned} \tag{8}$$

where  $P_{ij}(\boldsymbol{\tau})$  is the probability that household  $i$  chooses alternative  $j$  under TTR  $\boldsymbol{\tau}$  (according to expression (2) and  $T_{ij}(\boldsymbol{\tau})$  is the net tax paid by household  $i$  when choosing alternative under TTR  $\boldsymbol{\tau}$ . The constraint requires that the total expected net tax revenue be greater than (or equal to) a given amount  $R$ . Note that problem (8) assumes that the households are maximizing their utility functions, since the arguments of  $W$  are the (comparable money-metric) maximized utilities. The problem is solved with a numerical procedure. Given a vector of parameters  $\boldsymbol{\tau}$ , the microeconomic model simulates  $\mu_i(\boldsymbol{\tau})$ ,  $P_{ij}(\boldsymbol{\tau})$  and  $T_{ij}(\boldsymbol{\tau})$  for  $i = 1, \dots, H$  and  $j = 1, \dots, M$ . An optimization algorithm iterates the above simulation updating the value of  $\boldsymbol{\tau}$  until  $W$  cannot be further improved.<sup>13</sup>

***From the “primitives” to the optimal polynomial TTRs.*** The analytical optimal taxation identifies general TTR as a function of generic exogenous parameters  $\boldsymbol{\pi}$  called “primitives”, i.e. fundamental exogenous characteristics of the economy:  $TTR = f(\boldsymbol{\pi})$ . Examples of this result are expressions (10) and (11) of Appendix A. In order to specify the optimal TTR for a specific country  $c$ , the analytical approach imputes to the primitives the values  $\pi_c$  in order to get  $TTR_c = f(\boldsymbol{\pi}_c)$ . With the computational approach, we can follow the inverse path. First, we identify  $\boldsymbol{\tau}_c = \varphi(z_c)$ ,  $c = 1, 2, \dots, T$ , for  $T$  countries. where typically the optimal  $\boldsymbol{\tau}_c$  depends on characteristics  $z_c$  and  $\varphi$  is a computational procedure instead of a closed form function  $f$ . As a further step we can retrieve a mapping  $(\boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \dots, \boldsymbol{\pi}_T) \rightarrow (\boldsymbol{\tau}_1, \boldsymbol{\tau}_2, \dots, \boldsymbol{\tau}_T)$ . We will present a simple example that uses regression analysis. We consider the following “primitives”.

---

<sup>13</sup> In order to locate a global maximum, we partition the parameter space and try different starting values  $\boldsymbol{\tau}^0$ .

- 1) **Kolm's  $k$** . The inequality aversion parameter  $k$ , multiplied by 100. As a matter of fact, we have six different values of  $k$  for each one of the six countries, which makes 36 observations.
- 2) **Productivity**. The current average monthly taxable household income, as a measure of productivity.
- 3) **Extensive Elasticity**. The current own wage elasticity of labour supply (Table 7).
- 4) **Intensive Elasticity**.
- 5) **Budget**. The current monthly net tax revenue to be attained in order to satisfy the public budget constraint.<sup>14</sup>

We characterize the optimal TTRs with:

- 1)  $\tau_0$ . It is the universal basic income
- 2)  $100(1-\tau_1)$ . This is the percentage Leading Tax Rate. The definition is motivated by the fact that the other tax parameters  $\tau_2$ ,  $\tau_3$  and  $\tau_4$  are very small and have a sensible effect only on high levels of taxable income.
- 3) Local progressivity (MTR/ATR) at taxable income = 10000.
- 4) Local progressivity (MTR/ATR) at taxable income = 100000.

These are by far the most important features of the polynomial optimal TTRs. Moreover, notice that the higher  $\tau_0$  and the lower  $1-\tau_1$ , the larger is the range of taxable income with negative net taxes, therefore the ratio  $\tau_0 / (1-\tau_1)$  can be interpreted as an index of global progressivity. We estimate the regressions of the two characteristics of the optimal TTRs against the four “primitives”. The results are shown in Table 1 and commented in the next Section 4. The identification of the mapping (“primitives”)  $\rightarrow$  (features of the optimal TTR) can be useful in many different circumstances. For example, it can be used to suggest direction for reforms in countries where microdata are absent or not sufficiently rich, or to forecast the future need for reforms in view of expectations upon future changes in some of the “primitives”.

---

<sup>14</sup> It might be argued that “primitives” 2 – 5, are not really primitives, since they are also determined by the current TTRs. This is true, but it is not really relevant. We interpret our analysis as conditional upon the current economy.

#### 4. Results

The results of our exercise are documented by Table 1-2 and by Graphs 1a – 6b and 7 – 14. As a general synthesis: with the exception of Luxembourg, the optimal polynomial TTRs are welfare-superior to the current TTR for most values of Kolm's  $k$ ; optimal basic income is larger than the current expenditure on income maintenance, however it can also be smaller in some cases; optimal MTRs are almost constant for a large range of values of taxable income; the optimal TTRs lead to lower poverty; the distribution of welfare gains or losses across demographic groups is very heterogeneous across the different countries. Hereafter we provide more detailed comments and illustrations.

*The optimal TTRs.* Table 1 reports the parameters of the polynomial optimal TTRs, the polynomial approximation to the current TTRs and the welfare gain of the optimal TTRs with respect to the (real) current TTR. In all the countries, the shape of the optimal TTRs is dominated by  $\tau_1$ , while the other parameters are very small and might except some influence only at very large taxable incomes (e.g. above 2000 euro a year). As a consequence, the optimal TTRs are very close to a flat tax, with (almost) constant MTR equal to  $1-\tau_1$ . In contrast, the polynomial approximation to the current TTRs features important non-linearities, as it is revealed in particular by the values of parameters  $\tau_1$  and as compared to the corresponding parameters of their polynomial optimal TTRs. Parameter  $\tau_0$  is the monthly universal basic income (or guaranteed minimum income according to the NIT interpretation) for a one-person household. For a N-person household it must be multiplied by  $N^{1/2}$ . We also report the value of  $\tau_0$  in the polynomial approximation of the current TTRs. It is not strictly comparable to the optimized values of  $\tau_0$ , since the last ones are attained by universal and unconditional transfers to be received with certainty, while the former is an expected value across the population of various - mostly means-tested, contingent and categorical - transfers. It makes sense, however, to interpret  $\tau_0$  as a measure of the basic current expenditure in income support policies from view-point of the public budget constraint. In this perspective, one can judge that the current policies are more or less cost-effective than those dictated by the optimized rules. In France and Luxembourg, the current income support policies appear to be “too costly”: a less expensive universal and unconditional basic income would attain a higher Social Welfare (for  $k <$



0.125 in France and for  $k < 0.05$  in Luxembourg). The opposite holds in Germany and Italy (for  $k > 0.05$ ), Spain (for all considered value of  $k$ ) and the United Kingdom (for  $k \geq 0.05$ ). The main features of the optimal polynomial TTRs are also illustrated in the Graphs 1a – 6b. The effects of the other parameters  $\tau_1, \dots, \tau_4$  on the shape of the TTRs are illustrated by the Graphs 1a – 6a and 1b – 6b, which respectively represent MTR and ATR as functions the household total taxable income. It appears that in all the countries, the optimal MTR is almost constant over a large range of values of taxable income. This happens because, as we have seen in Table 1, the parameters  $\tau_2, \dots, \tau_4$  are very small, so that the MTR, except for very large taxable incomes, reduces to  $1 - \tau_1$ . The conclusion on this point is that, within the (flexible) TTR class considered, a certain degree of progressivity is more efficiently attained by an unconditional and universal Basic Income with non-distortive MTRs rather than by (increasing and distortive MTRs. We also represent the MTR of the polynomial approximation to the current TTR, which show striking differences both between the countries and with respect to the optimal polynomial TTRs. The optimal polynomial TTRs essentially feature a universal basic income with a (almost) flat tax or a negative income tax with a transfer reduction rate equal to the (almost) flat tax above the exemption level. It must be stressed that the “current” MTRs represented in the Graphs in general do not correspond to the formal current MTRs. They represent the effective marginal change in paid taxes for a household with average size. The current systems in France and Luxembourg appear to envisage relatively generous income support policies at low or zero income followed by very high implicit marginal benefit reduction rates. The optimal rules suggest less expensive (although universal and unconditional) income support and a longer and smoother phase-out. Germany envisages an expensive current income support policies and yet a slowly increasing MTR on low incomes. In Italy and Spain, the current MTRs are first steeply increasing up to taxable incomes around 100000 and then decreasing. The recent decades have witnessed a big interest in policies like In-work benefits or Tax Credits, which essentially imply negative effective MTRs on some range of (typically low) incomes. As a matter of fact, In-Work Benefits and Tax Credits are two pillars of the evolution of traditional income support policies to face the new scenario mentioned in the Introduction. Our results do not confirm a better performance of this type of policies: negative MTRs on low taxable incomes never emerge as elements of the optimal polynomial TTRs (although there are negative average net taxes on low incomes. However, it must be remembered

that these policies have received a favourable evaluation mostly due to their effects on labour supply incentives, while our evaluation criterion is a Social Welfare index. Recent contributions adopting the analytical approach (e.g. Saez 2002) have argued that a high participation elasticity of labour supply tends to favour policies like In-Work Benefits or Tax-Credits. In our results – as we will see later on when commenting the “mapping” results - a high participation elasticity favours instead a large Basic Income. The implication seems to be that unconditional and universal basic income reduces distortions more effectively than lower initial tax rates in low incomes.

Given that the optimal MTRs are very close to a constant, the ATRs (Graphs 1b – 6b) are useful to show the level and type of progressivity implied by the various TTRs in the different countries.<sup>15</sup> If the Social Welfare criterion ignores inequality effects (i.e.  $k = 0.00$ ), in all the countries the optimal TTR – as compared to the current TTR – is more progressive on low levels of taxable income and more progressive on middle or high taxable incomes. The opposite happens with  $k=0.15$ . This holds in general, although in Luxembourg and Germany the ATRs are very close for different values of  $k$ , where the ATR behaves approximately in the same way whatever the value of  $k$ . For  $k = 0.075$ , the optimal ATR is closer to the Current one, but less progressive on middle and high incomes in France and Italy. Judging from the point-of view of the optimal TTRs, the main flaws of the current TTRs – and the corresponding implicit reform recommendations – can be summarized as follows. France: current income support too expensive, current MTR too high on low and high income. Germany: current MTR too low on low incomes and too high on high incomes. Italy: current income support not cost-effective, current MTR too low on low and high incomes and too high on middle incomes. Luxemburg: nothing to complain about, except maybe current income support not cost-effective if  $k = 0.00$ . Spain: current income support too low, current MTR on low incomes too low. The United Kingdom: current MTR too low on low incomes and too high on middle incomes.

**Welfare effects.** The Social Welfare gains (SWG) and the Equality Gains (EG) due to the optimal polynomial TTRs by country and Kolm’s  $k$  are reported in Table 1, in the last two rows of each

---

<sup>15</sup> A simple index of progressivity is MTR/ATR.

country-specific section.<sup>16</sup> Note that the current Social Welfare values are those attained the actual current TTRs, not by the polynomial approximations to them. For most countries and most values of  $k$ , the optimal polynomial TTR is social welfare superior to the current TTR. This result holds in France and Italy for  $k < 0.15$ , in Luxembourg for  $k < 0.05$ , in Germany and Spain for  $k \geq 0.075$  and in the United Kingdom for  $k \geq 0.05$ . What happens is that the polynomial optimal TTRs are mainly disequalizing in France, Italy and Luxembourg but equalizing (for a majority of  $k$  values) in Spain and in the United Kingdom. As consequence, higher values of  $k$  – i.e. higher costs of inequality – tend overcome the efficiency effect in the former group of countries and strengthen it in the latter one. These results are also due to the efficiency effect (i.e. SWG – EG), which decreases with  $k$  in the first group of countries while it increases in the second one.

Besides the overall Social Welfare effects, we can identify the winners and the losers of a new TTR. For different demographic groups (couples, single male and single females) we have computed the CMU (Section 3) under the current TTR and under the optimal TTRs for  $k = .075$ .<sup>17</sup> Graphs 7 – 12 show the average CMU gains in different demographic groups, by decile (1-3, 4-7, 8-10) of current CMU<sup>18</sup>. The graphs show an extreme heterogeneity across countries, demographic groups and deciles. Depending on the country, some groups and/or some deciles are penalized by the optimal polynomial TTRs. System like NIT+FT are typically expected to penalize middle deciles. In our results this is indeed the case except for France and Germany. There are however important differences among countries and demographic groups. In view of implementing the optimal rules, most of these problems might probably be moderated by a country-specific design of the equivalence scale applied to the basic income and/or to other parameters of the Optimal TTRs.

***Economic effects.*** Graphs 13 and 14 represent the % change in disposable income and the Poverty Gap Index respectively, by country and Kolm's  $k$ . The two graphs illustrate a manifestation of the efficiency-equality trade-off. Disposable income increases as long as  $k \leq 0.05$ , with the exception

---

<sup>16</sup> The Equality Gain is computed as -Inequality Gain, where Inequality is defined by expression (7). Note that the Efficiency Gain can also be retrieved as SWG – EG.

<sup>17</sup> The shape of results is similar for different values of  $k$ .

<sup>18</sup> We prefer CMU instead of the ordinal criterion based only on the proportion of winners, since CMU also accounts for the size of the gains or losses.

of Germany. With  $k > 0.05$  it keeps increasing in France and Luxembourg, while it decreases in Germany, Italy and the United Kingdom. The Poverty Gap Index in Graph 14 shows a similar pattern. The aggregate effects on labour supply (not reported) incomes are small, although we might observe more significant changes for different demographic groups or income levels (not reported in this paper).

***The “primitives” and the optimal TTRs.*** Table 1 shows the results of inferring a general rule that links the “primitives” to the optimal TTRs, i.e. it presents the results of the analysis explained in Section 3. We have well-defined results on Basic Income ( $\tau_0$ ): all the coefficients are significant at standard levels with the expected sign. Kolm’s  $k$ , Productivity and Elasticity (both extensive and intensive) elasticities favour a higher Basic Income. A stricter Budget require a lower Basic Income. Among the above results, the surprising one is the effect of elasticities. A possible explanation is that Basic Income, as compared to means-tested policies does not suffer from poverty-traps, therefore its relative advantage is greater the more elastic is household behaviour. Kolm’s  $k$  and Extensive elasticity respectively favour a lower and a higher Leading Tax rate Productivity: the former result, taken together with  $k$ ’s effect on Basic Income, seems to mean that more egalitarian social preferences favour a higher Basic Income rather than higher taxes; the latter result might mean that less distortions are better achieved with a universal basic income than with lower taxes. The regressions on local indices of progressivity are less informative. Kolm’s  $k$  asks for more progressivity (measured as MTR/ATR) both at low and high levels of taxable income. There is a positive effect of (intensive) Elasticity upon Local progressivity at taxable income = 10000. At this value, we are just out the negative tax area, so it probably makes sense that a higher Elasticity (which favours a higher Basic Income) starts asking for more progressivity in order to raise tax revenue. Just as an illustrative example, let’s us imagine that we want to propose a common TTR to all the countries, based on the averages of the “primitives”. Let us also suppose that social preferences are such that  $k = 0.075$ . Then the basic income and the leading tax rate of the common polynomial optimal TTR would be 456 monthly euros (for one-person household) and 29.8% respectively. It is close to the optimal TTR in Germany for  $k = 0.05$ .

## 5. Concluding remarks

As a response to a new labour market scenario emerged during the last decades, featuring a larger heterogeneity in household behaviour, preferences and needs and implying adverse labour incentives and raising costs for the welfare system, many countries have introduced reforms with more sophisticated design of mean-tested and categorical tax-transfer policies. In this exercise we have followed a different policy direction, inspired by simplicity and universality. We have adopted the perspective of optimal taxation. Two main approaches to empirical optimal income taxation have been used so far in the literature: the analytical and the computational approach. In this paper we develop a version of the computational approach that combines microeconomic modelling, microsimulation, numerical optimization and social welfare evaluation in a consistent way. We consider the class of 4<sup>th</sup> polynomial TTRs, i.e. a generic rule that represents total household disposable income as a 4<sup>th</sup> degree polynomial function plus a constant. We adopt the Kolm's social welfare function. A specific TTR is defined by the parameter vector containing the four coefficients of the polynomial and the constant. We identify optimal TTRs for different degrees of social inequality aversion and compare them to the current rules in six European countries. For most countries and most values of the inequality aversion parameter, the optimal polynomial rules provide a higher social welfare than the current ones.

The class of TTRs considered as candidates for welfare optimality, although flexible, is extremely simple. It is applied to total taxable household income, irrespective of the source of income. It does not depend household's socio-economic characteristics, with the exception on the number of household member that affects the basic income transfer. It is of course quite possible that we might do better by taking households' heterogeneity into account when designing the optimal TTR. However, finely categorized and means-tested TTR bear administration and political costs (complex forms to file out, monitoring, political manipulation, lack of transparency, conflict resolution etc.) that are certainly less important or even not existent in simple and universalistic TTRs. Our research goal consists of investigating whether a very simple and universalistic design is able to outperform, social welfare-wise, the current system: in most countries it does.

The results suggest some common features in all the countries. The optimal polynomial TTRs envisage a universal unconditional basic income with a (almost) flat tax or, depending on the interpretations, a (almost) flat negative income tax with guaranteed minimum. The profiles of the

MTRs are pretty similar in different countries and definitely flatter than under the current TTRs. This result holds for all the considered countries, despite the fact that the polynomial TTR class is very flexible and the heterogeneous responses allowed by the microeconomic model might induce very different shapes of the optimal TTRs. These results confirm those of Islam and Colombino (2018). This is remarkable, since Islam and Colombino (2018) only compare the NIT+FT rule to the current TTR, while the polynomial class considered in this paper is very flexible. The social welfare gains due to the optimal polynomial TTRs are admittedly small. However, there are other benefits – not accounted for in this exercise – that presumably can be provided by that type of TTR, such as simplicity, lower administration costs and reduced opportunities for corruption and political manipulation.

Despite the above common features, we can see large differences between the levels of the basic income and the values of the MTRs under the optimal polynomial TTRs in the different countries. They depend indeed on various characteristics of the population and the economic environment. The differences of optimal TTRs in different countries, therefore, call for a further step. An explanation of these differences among countries requires to identify a general relationship between the basic (“primitive”) characteristics of the economy and the features of the optimal TTRs. Actually, this is the direct result of analytical optimal taxation (i.e. expressions (10) or (11) in Appendix A). We can come close to a similar result by replacing the analytical solution with microsimulation and numerical optimization. Even with a limited number of countries, we are able to identify some significant effects of “primitives” (Kolm’s  $k$ , Productivity, Extensive and Intensive Elasticities, Public budget constraint) upon four characteristics of the optimal TTRs (Basic Income, Leading tax rate, Local Progressivity at low and at high taxable income). Overall, the results are not at odds with the typical results obtained by the analytical approach. However we also have specific results that seem to stem from the resulting social preference for universal basic income and the (almost) flat tax. This supports a complementary role of the computational approach. While the analytical solutions might indicate promising reform directions, the computational approach in principle is able to account for more realistic features of the economies.

# Tables

Table 1. “Current” and optimal TTRs								
		“Current”	k = .00	k = .05	k = .075	k = .10	k = .125	k = .15
France	$\tau_0$	603.27	61.43	181.72	265.57	367.97	453.77	592.02
	$\tau_1$	0.52	0.93	0.88	0.84	0.80	0.76	0.70
	$\tau_2$	$3.01 \times 10^{-6}$	$-0.0000 \times 10^{-6}$	$0.0106 \times 10^{-6}$	$0.013 \times 10^{-6}$	$0.011 \times 10^{-6}$	$0.0098 \times 10^{-6}$	$0.0097 \times 10^{-6}$
	$\tau_3$	$-1.51 \times 10^{-11}$	$0.0004 \times 10^{-11}$	$0.0062 \times 10^{-11}$	$0.015 \times 10^{-11}$	$0.005 \times 10^{-11}$	$0.0064 \times 10^{-11}$	$0.0053 \times 10^{-11}$
	$\tau_4$	$0.20 \times 10^{-16}$	$0.0009 \times 10^{-16}$	$0.008 \times 10^{-16}$	$0.014 \times 10^{-16}$	$0.017 \times 10^{-16}$	$0.0098 \times 10^{-16}$	$0.0051 \times 10^{-16}$
	SWG	-	159.23	89.63	53.17	16.37	-8.6	-45.47
	EG	-	0	-20.31	-20.64	-12.4	-1.86	20.3
Germany	$\tau_0$	427.9	427.9	454.23	554.48	600.91	600.9075	701.7375
	$\tau_1$	0.71	0.71	0.69	0.66	0.64	0.64	0.61
	$\tau_2$	$0.0016 \times 10^{-6}$	$0.003 \times 10^{-6}$	$0.001 \times 10^{-6}$	$0.0001 \times 10^{-6}$	$0.001 \times 10^{-6}$	$0.002 \times 10^{-6}$	$0.002 \times 10^{-6}$
	$\tau_3$	$0.0009 \times 10^{-11}$	$0.005 \times 10^{-11}$	$-0.002 \times 10^{-11}$	$0.001 \times 10^{-11}$	$0.001 \times 10^{-11}$	$0.001 \times 10^{-11}$	$0.001 \times 10^{-11}$
	$\tau_4$	$0.0022 \times 10^{-16}$	$0.010 \times 10^{-16}$	$0.004 \times 10^{-16}$	$0.001 \times 10^{-16}$	$0.001 \times 10^{-16}$	$0.004 \times 10^{-16}$	$0.002 \times 10^{-16}$
	SWG	-	-7.48	-5.00	41.05	71.39	87.65	155.27
	EG	-	0	-2.02	8.51	23.68	39.94	78.61
Italy	$\tau_0$	217.24	98.99	177.28	236.69	270.67	350.15	417.72
	$\tau_1$	0.745	0.752	0.698	0.66	0.63	0.58	0.53
	$\tau_2$	$-1.98 \times 10^{-6}$	$-0.02 \times 10^{-6}$	$-0.01 \times 10^{-6}$	$0.002 \times 10^{-6}$	$0.005 \times 10^{-6}$	$0.0002 \times 10^{-6}$	$0.0008 \times 10^{-6}$
	$\tau_3$	$0.69 \times 10^{-11}$	$0.04 \times 10^{-11}$	$0.01 \times 10^{-11}$	$0.004 \times 10^{-11}$	$0.004 \times 10^{-11}$	$0.0000 \times 10^{-11}$	$-0.0004 \times 10^{-11}$
	$\tau_4$	$-0.07 \times 10^{-16}$	$-0.02 \times 10^{-16}$	$-0.01 \times 10^{-16}$	$-0.0000 \times 10^{-16}$	$0.003 \times 10^{-16}$	$0.0005 \times 10^{-16}$	$0.0025 \times 10^{-16}$
	SWG	-	72.9	45.93	28.4	18.64	-1.35	-18.54
	EG	-	0	-3	-3.06	-2.97	0.13	3.16
Luxembourg	$\tau_0$	1469.68	615.73	680.78	746.64	809.23	858.74	926.17
	$\tau_1$	0.316	0.761	0.75	0.717	0.706	0.692	0.676
	$\tau_2$	$4.12 \times 10^{-6}$	$0.228 \times 10^{-6}$	$0.24 \times 10^{-6}$	$0.23 \times 10^{-6}$	$0.245 \times 10^{-6}$	$0.227 \times 10^{-6}$	$0.223 \times 10^{-6}$
	$\tau_3$	$-1.869 \times 10^{-11}$	$0.019 \times 10^{-11}$	$0.022 \times 10^{-11}$	$0.014 \times 10^{-11}$	$0.051 \times 10^{-11}$	$0.041 \times 10^{-11}$	$0.013 \times 10^{-11}$
	$\tau_4$	$0.25 \times 10^{-16}$	$0.07 \times 10^{-16}$	$0.017 \times 10^{-16}$	$0.075 \times 10^{-16}$	$0.008 \times 10^{-16}$	$0.009 \times 10^{-16}$	$-0.004 \times 10^{-16}$
	SWG	-	2.06	-35.63	-68.35	-62.63	-67.54	-66.94
	EG	-	0	-41.82	-43.04	-46.45	-40.26	-27.94
Spain	$\tau_0$	196.42	227.43	225.51	436.88	481.18	493.54	498.23
	$\tau_1$	0.97	0.81	0.81	0.65	0.61	0.60	0.60
	$\tau_2$	$-4.24 \times 10^{-6}$	$0.001 \times 10^{-6}$	$0.0005 \times 10^{-6}$	$-0.0012 \times 10^{-6}$	$0.001 \times 10^{-6}$	$0.0005 \times 10^{-6}$	$-0.0001 \times 10^{-6}$
	$\tau_3$	$2.05 \times 10^{-11}$	$0.001 \times 10^{-11}$	$.0008 \times 10^{-11}$	$-0.0008 \times 10^{-11}$	$0.0003 \times 10^{-11}$	$-0.0001 \times 10^{-11}$	$0.0000 \times 10^{-11}$
	$\tau_4$	$-0.36 \times 10^{-16}$	$0.001 \times 10^{-16}$	$0.0008 \times 10^{-16}$	$-0.003 \times 10^{-16}$	$0.001 \times 10^{-16}$	$0.0002 \times 10^{-16}$	$0.0000 \times 10^{-16}$
	SWG	-	-16.99	-13.87	10.65	13.83	21.35	27.17
	EG	-	0	2.17	13.88	20.25	25.47	30.29
United Kingdom	$\tau_0$	455.51	289.08	564.61	564.82	628.61	749.57	864.62
	$\tau_1$	0.66	0.80	0.61	0.61	0.57	0.48	0.40
	$\tau_2$	$1.717 \times 10^{-6}$	$-0.0002 \times 10^{-6}$	$0.0003 \times 10^{-6}$	$0.0003 \times 10^{-6}$	$0.001 \times 10^{-6}$	$-0.005 \times 10^{-6}$	$-0.001 \times 10^{-6}$
	$\tau_3$	$-2.008 \times 10^{-11}$	$0.0002 \times 10^{-11}$	$0.0001 \times 10^{-11}$	$0.001 \times 10^{-11}$	$-0.0006 \times 10^{-11}$	$0.002 \times 10^{-11}$	$-0.002 \times 10^{-11}$
	$\tau_4$	$0.505 \times 10^{-16}$	$0.0001 \times 10^{-16}$	$0.0001 \times 10^{-16}$	$0.0004 \times 10^{-16}$	$-0.0004 \times 10^{-16}$	$-0.003 \times 10^{-16}$	$-0.001 \times 10^{-16}$
	SWG	-	-18.37	37.42	37.81	48.34	65.19	77.19
	EG	-	0	0.81	1.14	1.24	0.64	-0.17



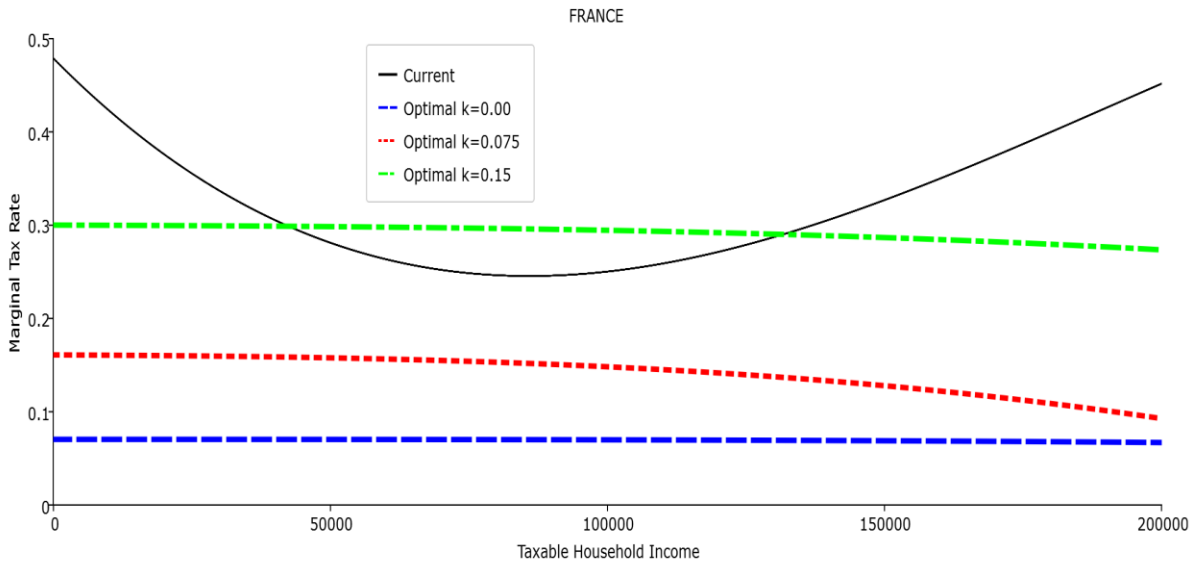
Note to Table 1: **SWG** = social Welfare Gain, **EG** = Equality Gain

	<b>Basic Income</b>	<b>Leading Tax Rate</b>	<b>Local Progressivity at taxable = 10000</b>	<b>Local Progressivity at taxable = 100000</b>
<b>Constant</b>	<b>-357.28</b> <i>-2.14</i>	-5.04 <i>-0.3</i>	-36.97 <i>-0.48</i>	<b>1.37</b> <i>3.57</i>
<b>Kolm's <math>k</math></b>	<b>25.44</b> <i>14.01</i>	<b>1.40</b> <i>9.95</i>	<b>1.51</b> <i>1.79</i>	<b>0.01</b> <i>3.38</i>
<b>Productivity</b>	<b>0.22</b> <i>11.29</i>	0.002 <i>1.48</i>	0.001 <i>0.135</i>	-0.00 <i>-0.71</i>
<b>Extensive Elasticity</b>	<b>117.35</b> <i>3.05</i>	<b>9.90</b> <i>3.31</i>	8.41 <i>0.47</i>	-0.06 <i>-0.67</i>
<b>Intensive Elasticity</b>	<b>253.55</b> <i>8.44</i>	-1.46 <i>-0.62</i>	<b>27.28</b> <i>1.96</i>	0.032 <i>0.39</i>
<b>Budget</b>	<b>-0.25</b> <i>-8.08</i>	-0.002 <i>-0.85</i>	0.001 <i>0.08</i>	-0.00 <i>-1.10</i>
<b>R<sup>2</sup></b>	0.83	0.88	0.25	0.31
<b>Standard error of the estimate</b>	53.69	0.04	24.91	0.12
t-values in italics below the estimates Bold estimates are significant at standard levels				

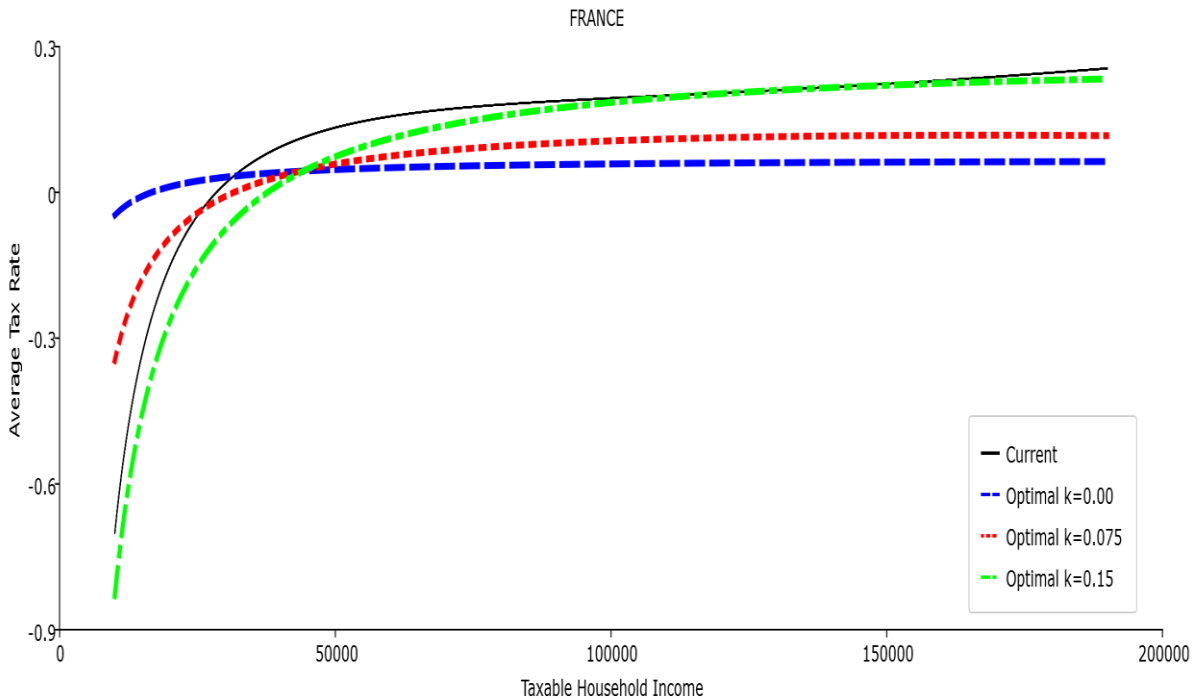
**Table 2. "Primitives" of the economy and characteristics of the optimal polynomial TTRs**

# Graphs

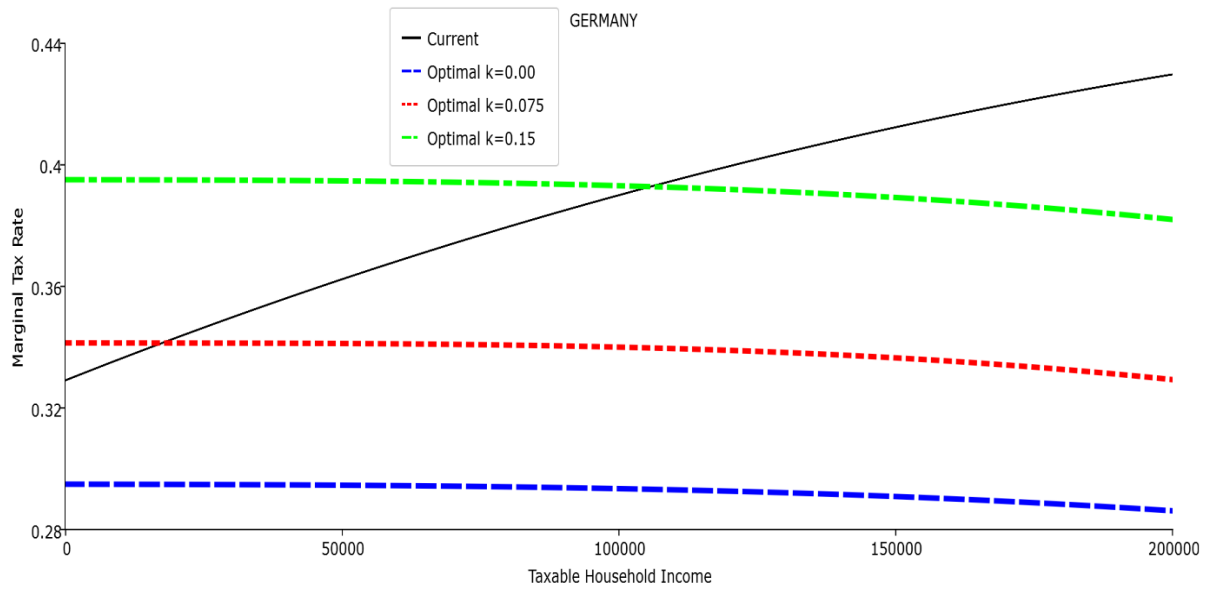
**Graph 1a. Marginal Tax Rate vs. Taxable Income. France**



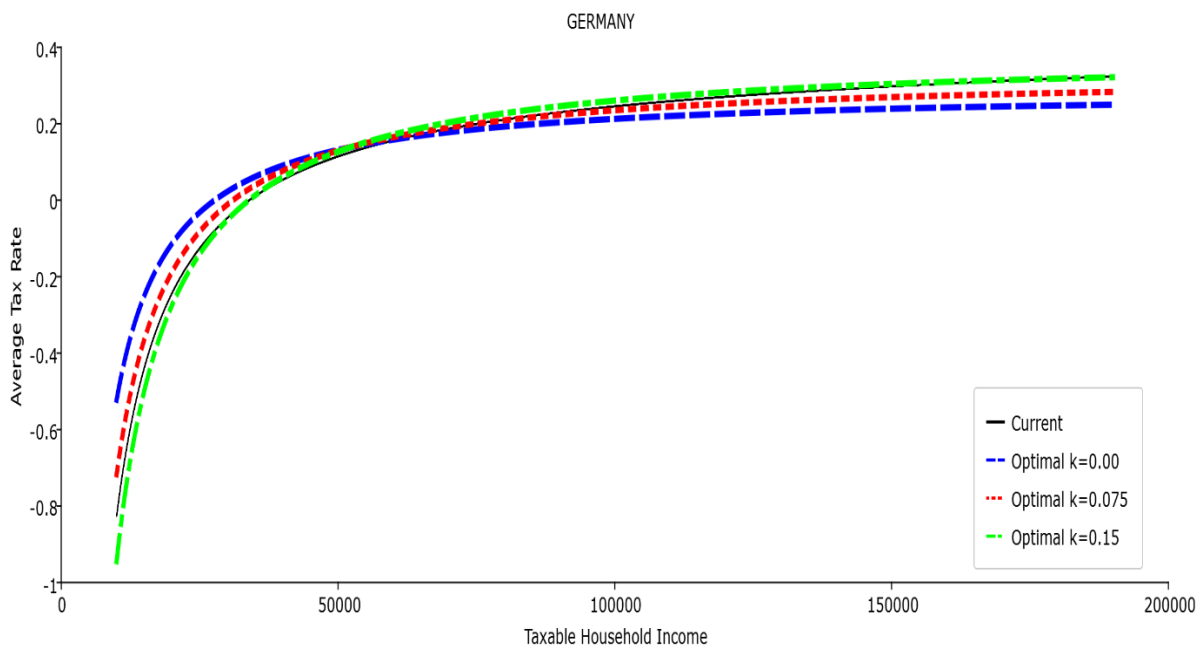
**Graph 1b. Average Tax Rate vs. Taxable Income. France**



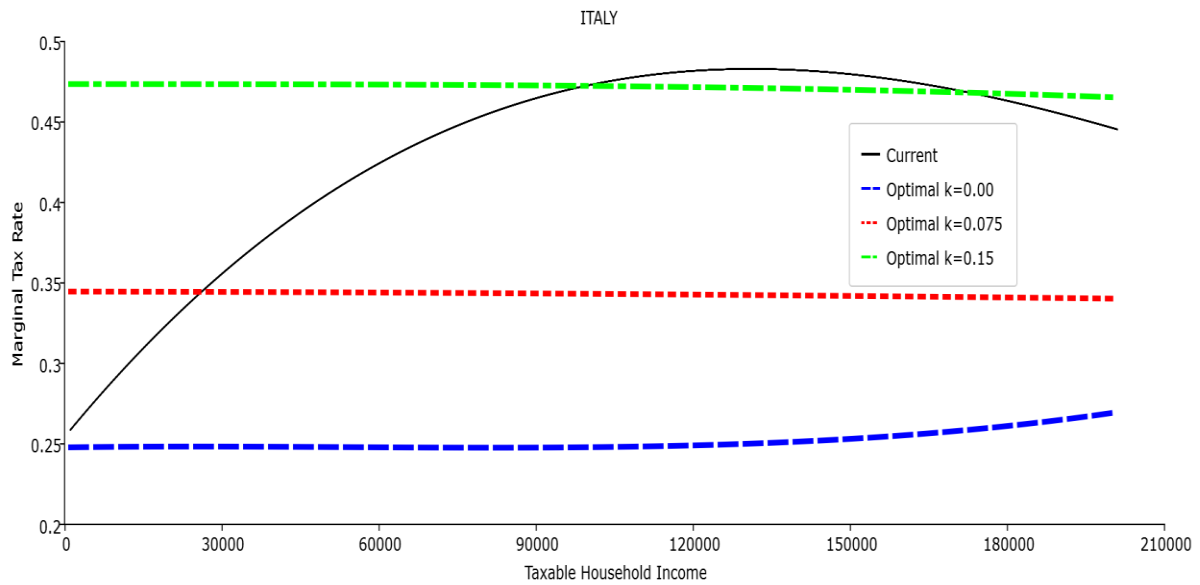
**Graph 2a. Marginal Tax Rate vs. Taxable Income. Germany**



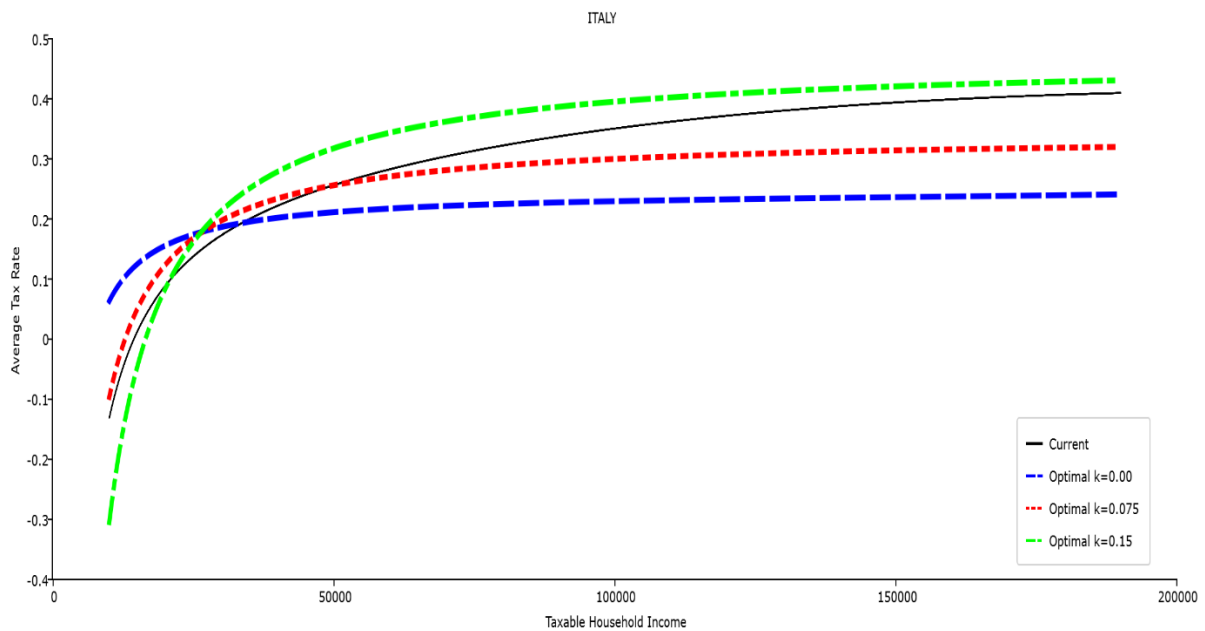
**Graph 2b. Average Tax Rate vs. Taxable Income. Germany**



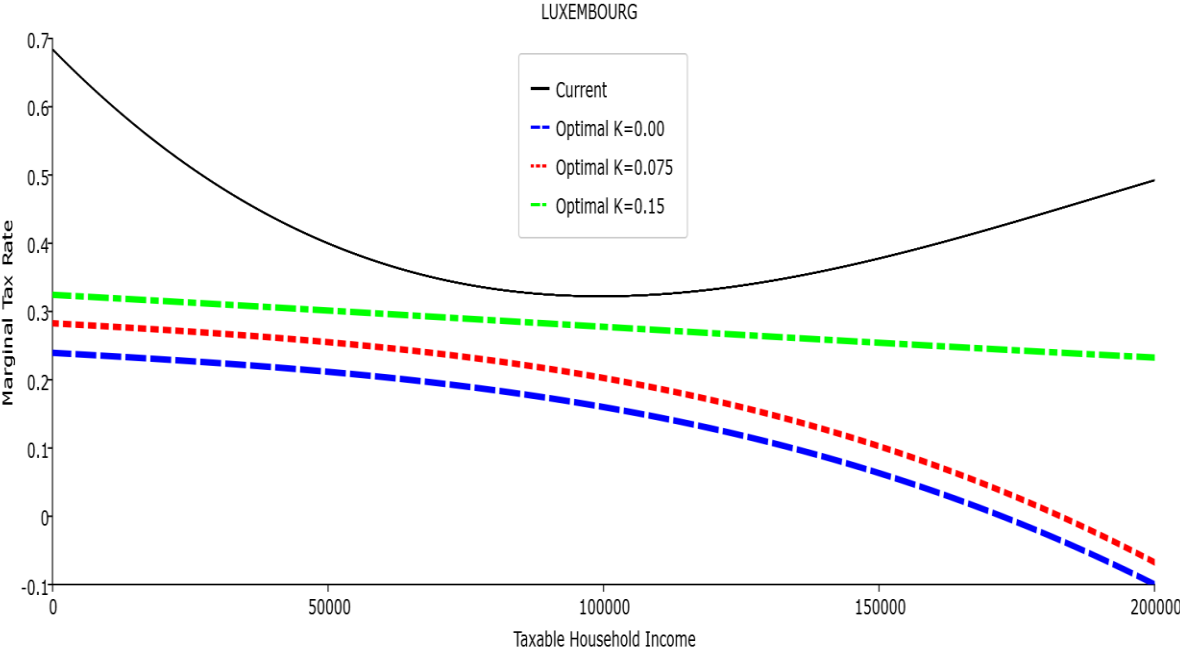
**Graph 3a. Marginal Tax Rate vs. Taxable Income. Italy**



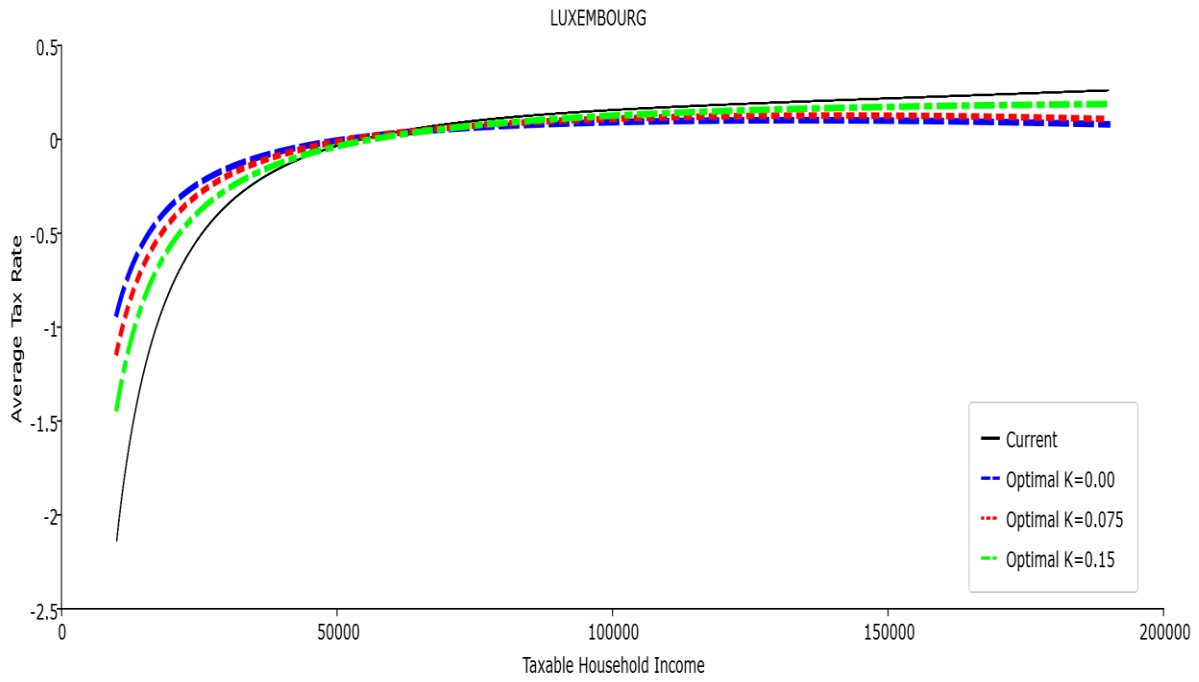
**Graph 3b. Average Tax Rate vs. Taxable Income. Italy**



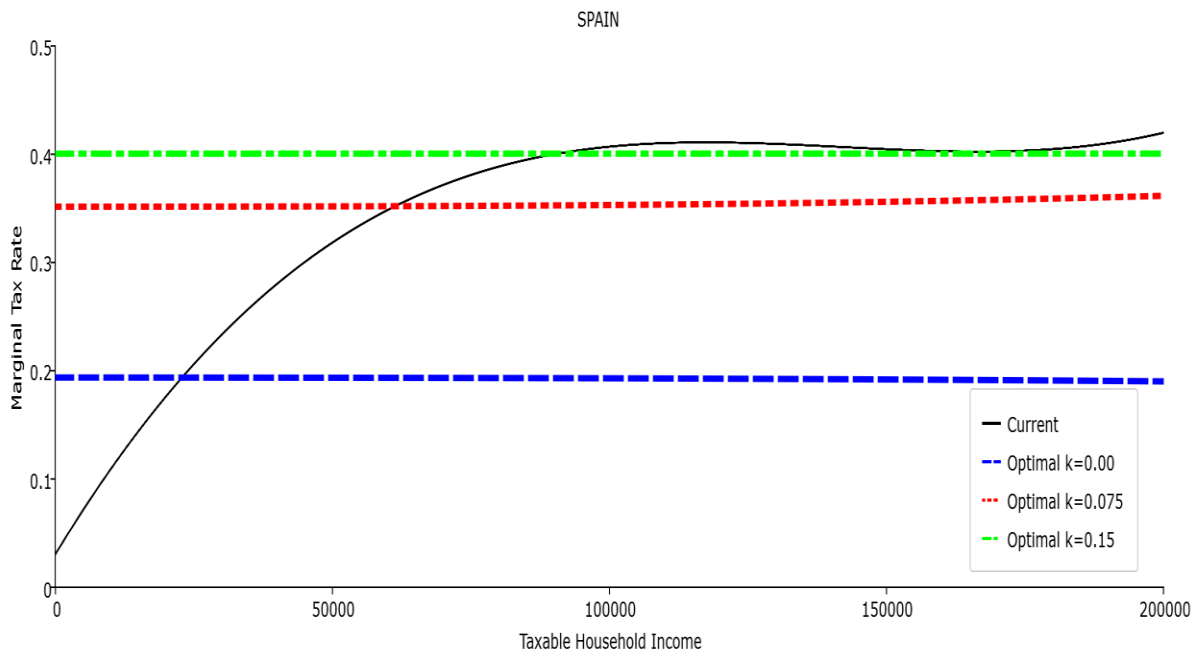
**Graph 4a. Marginal Tax Rate vs. Taxable Income. Luxembourg**



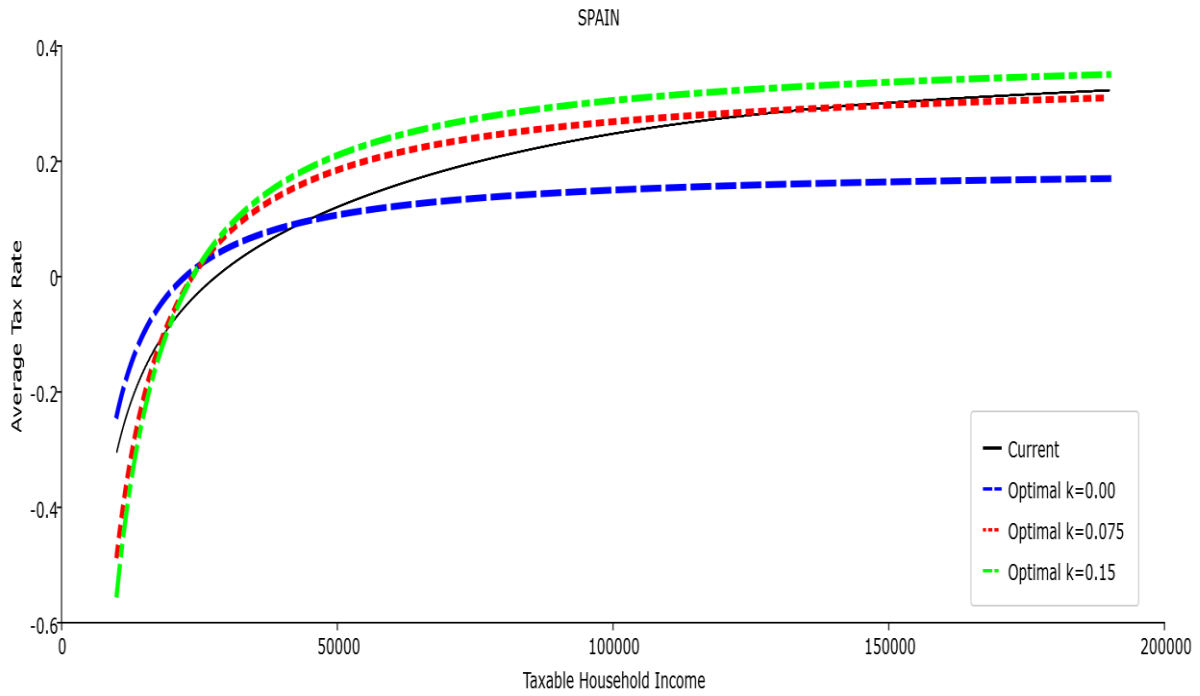
**Graph 4b. Average Tax Rate vs. Taxable Income. Luxembourg**



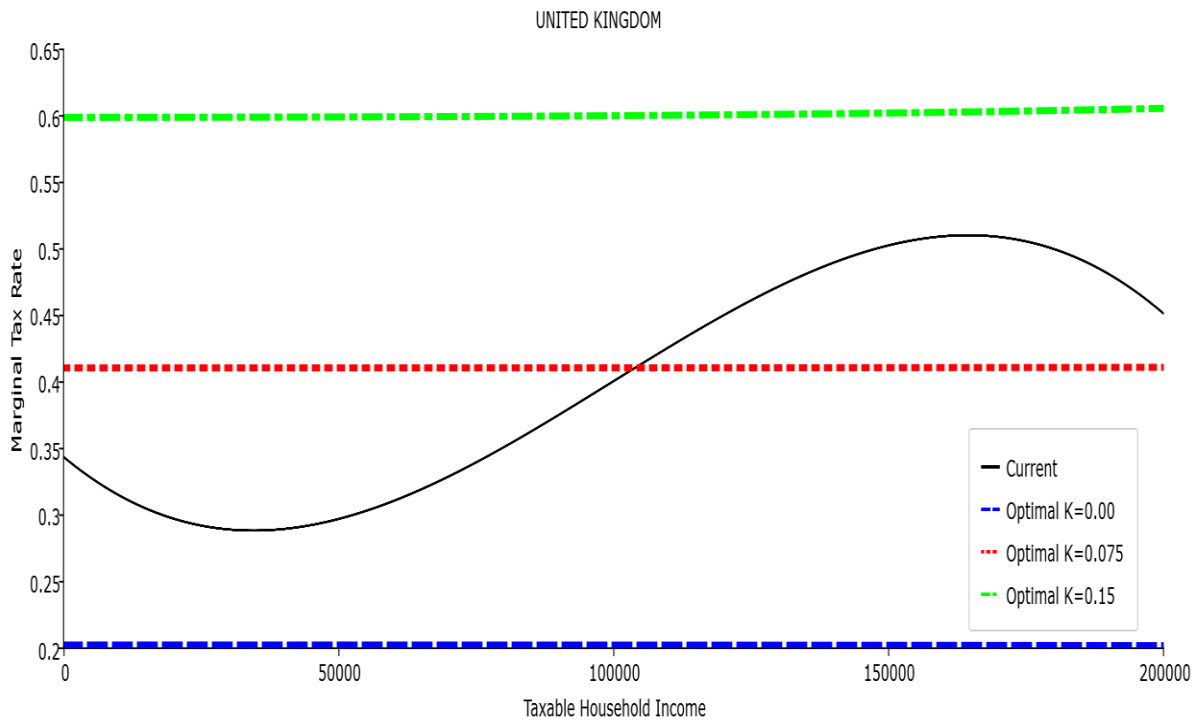
**Graph 5a. Marginal Tax Rate vs. Taxable Income. Spain**



**Graph 5b. Average Tax Rate vs. Taxable Income. Spain**

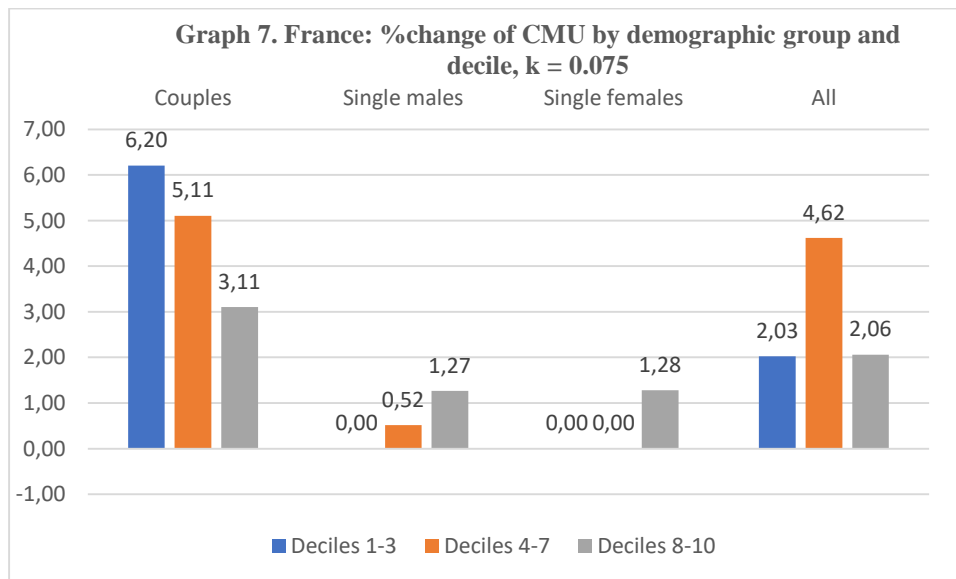
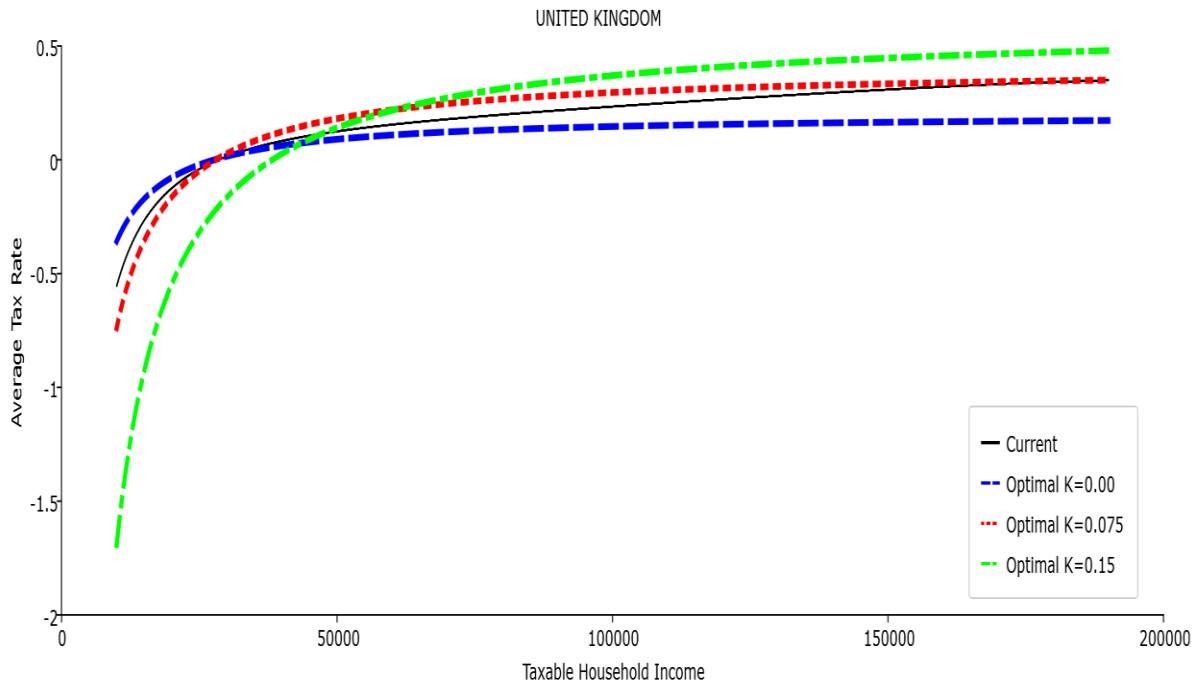


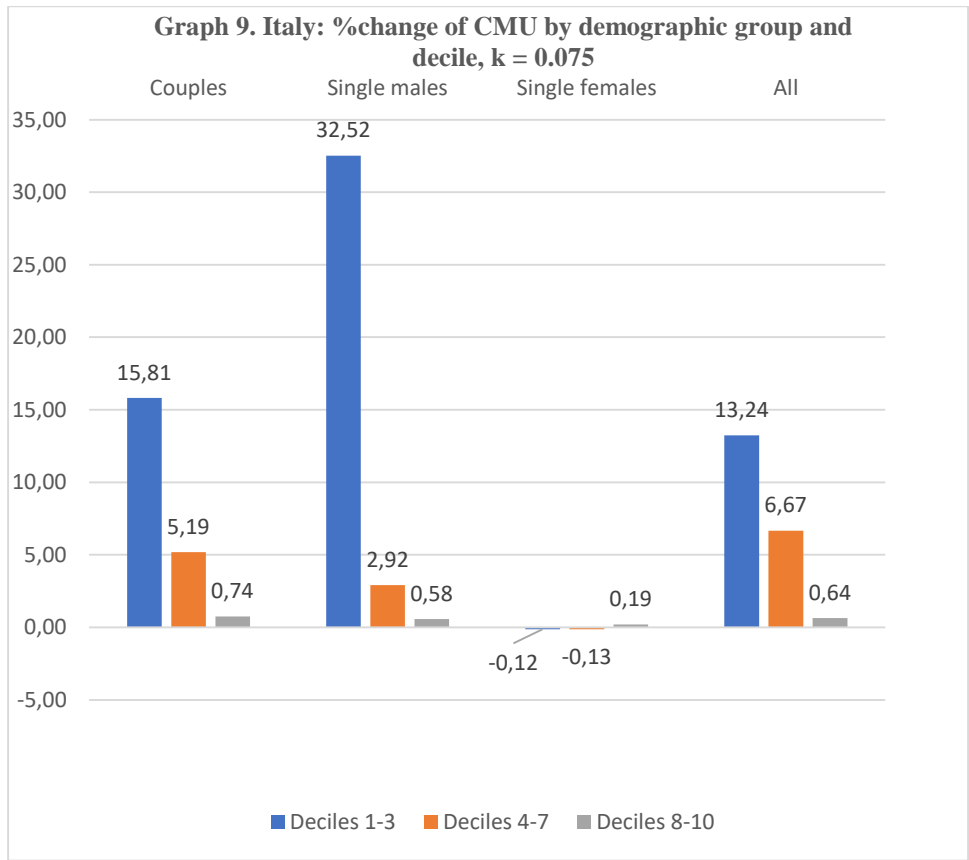
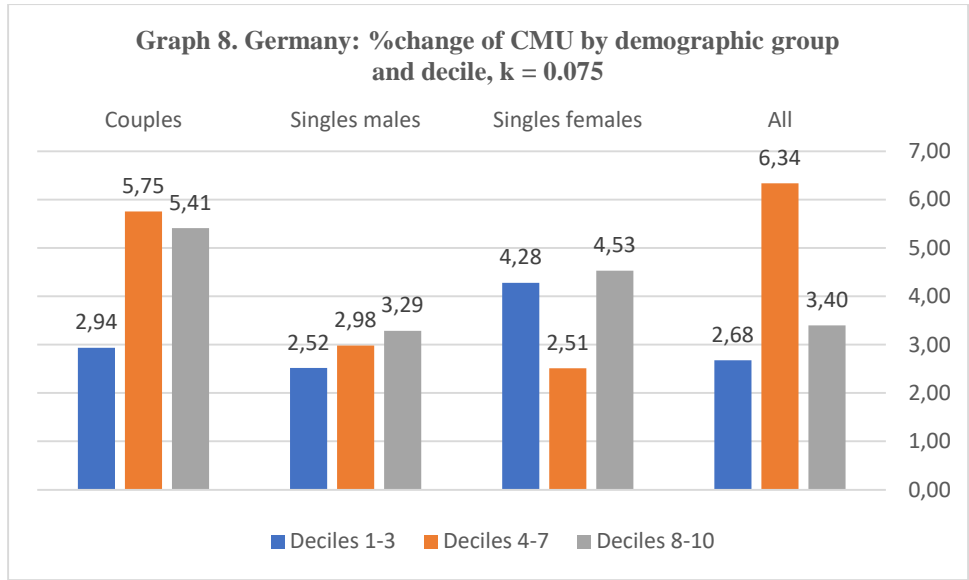
**Graph 6a. Marginal Tax Rate vs. Taxable Income. United Kingdom**

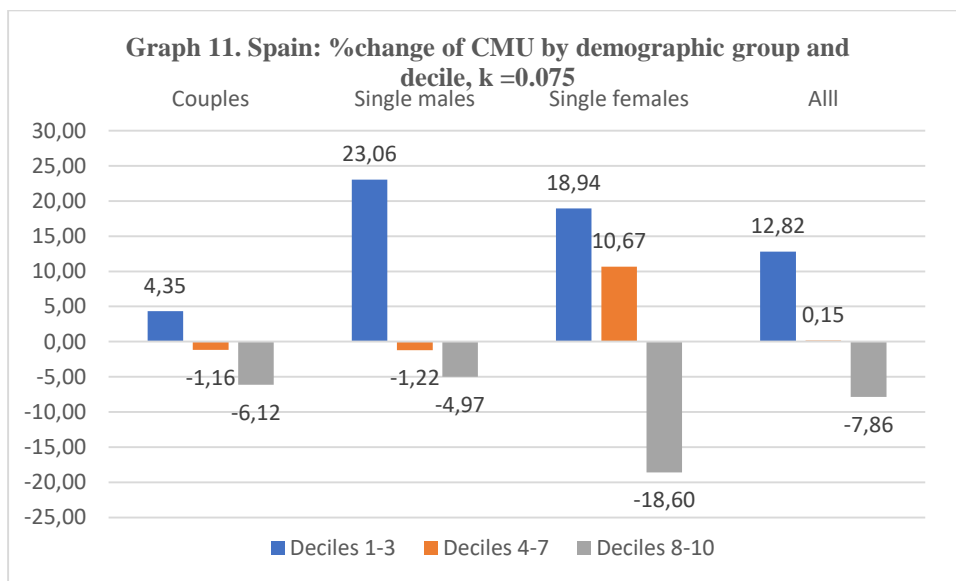
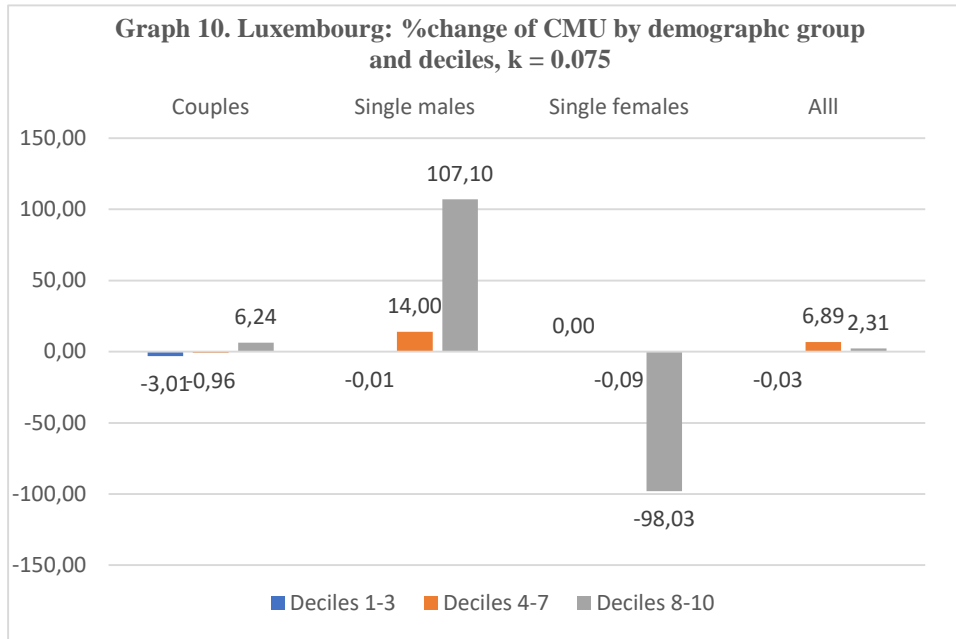




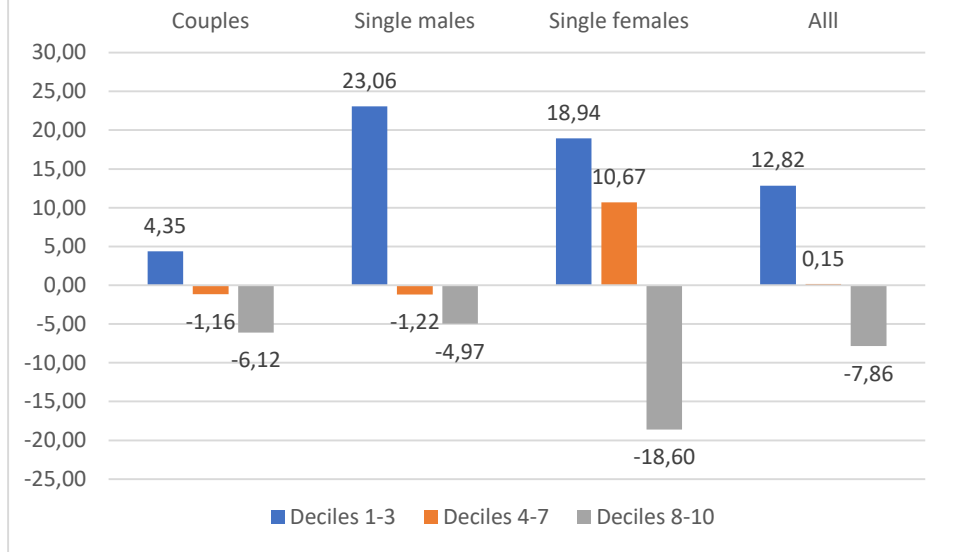
**Graph 6b. Average Tax Rate vs. Taxable Income. United Kingdom**



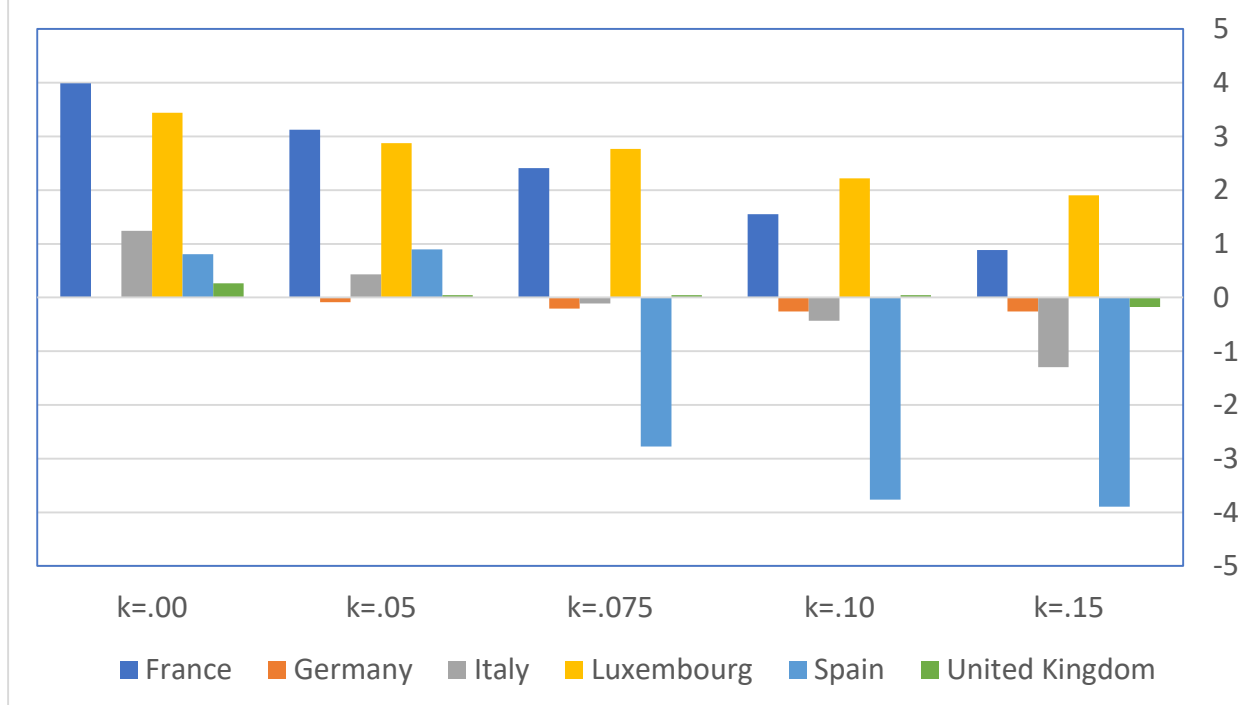




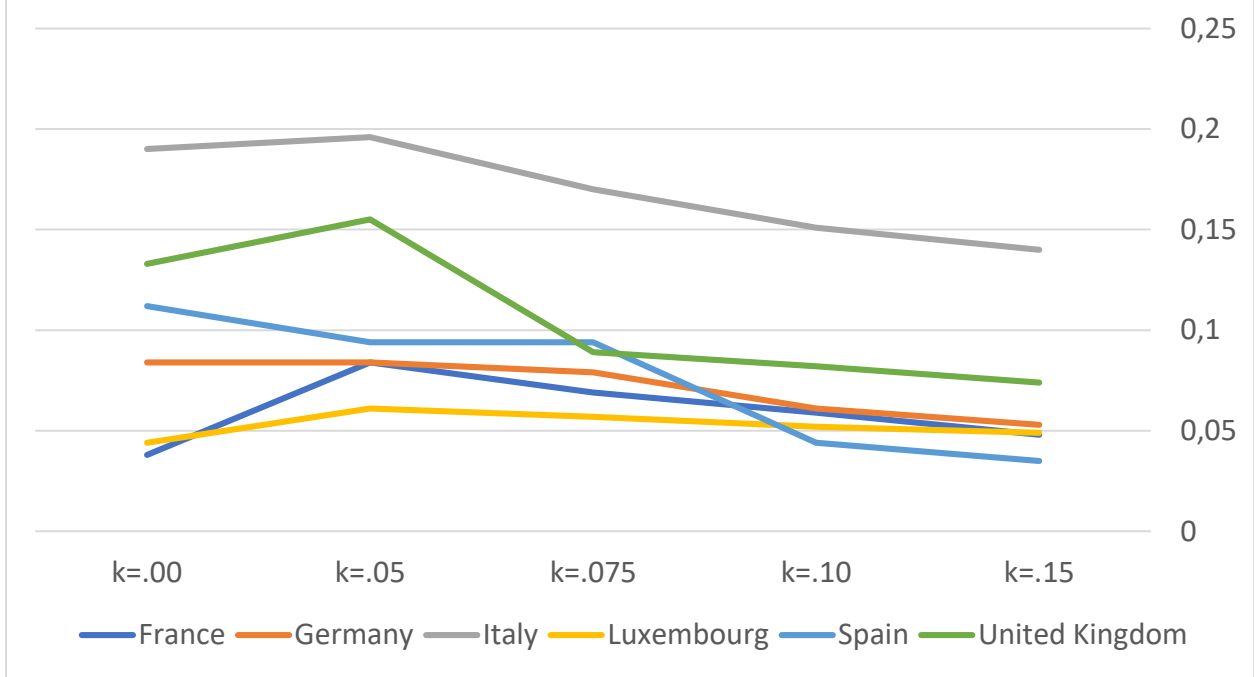
**Graph 12. The UK: %change of CMU by demographic group and decile, k = 0.075**



**Graph 13. %change of disposable income w.r.t. current values by country and Kolm's k**



Graph 14. Poverty Gap Index by country and Kolm's k



## 6. References

Aaberge, R. and U. Colombino (2014). Labour Supply Models, in C. O'Donoghue (Ed.), Handbook of Microsimulation Modelling, Contributions to Economic Analysis vol. 293, Emerald.

Aaberge, R. and U. Colombino (2013). Using a microeconomic model of household labour supply to design optimal income taxes. *Scandinavian Journal of Economics*, 115(2), 449–475.

Aaberge, R. and U. Colombino (2012). Accounting for family background when designing optimal income taxes: a microeconomic simulation analysis. *Journal of Population Economics*, 25(2), 741–761.

Aaberge, R. and U. Colombino (2006). Designing optimal taxes with a microeconomic model of household labour supply. IZA Discussion Papers 2468, Institute for the Study of Labor (IZA).

Aaberge, R. , Colombino U. and Strøm S. (2004). Do more equal slices shrink the cake? An empirical investigation of tax-transfer reform proposals in Italy. *Journal of Population Economics*, 17(4), pages 767-785, December.

Atkinson, A. B. (1996). *Public Economics in Action: The Basic Income/Flat Tax Proposal*. Oxford: Clarendon Press.

Atkinson, A. B. and A. Brandolini (2010). On Analyzing the World Distribution of Income, *World Bank Economic Review*, 24(1), 1-37.

R. M. Blank (2002). U.S. Welfare Reform: What's Relevant for Europe? CESifo Working Paper Series 753,

Blundell, R. & Shephard, A. (2012). Employment, hours of work and the optimal taxation of low-income families. *Review of Economic Studies*, 79(2), 481–510.

Brewer, M., Saez, E. and A. Shephard, (2008) Means-testing and tax rates on earnings (Mirrlees Commission Report), Institute for Fiscal Studies, London.

Bryan, J. (2005). Have the 1996 welfare reforms and expansion of the earned income tax credit eliminated the need for a basic income guarantee in the US? *Review of Social Economy* 63(4), 595–611.

Chetty, R. (2009). Sufficient Statistics for Welfare Analysis: A Bridge Between Structural and Reduced-Form Methods, *Annual Review of Economics*, *Annual Reviews* 1(1), 451-488.

Coda Moscarola, F., Colombino, U., Figari, F. and M. Locatelli (2020). Shifting taxes from labour to property. A simulation under labour market equilibrium, *Journal of Policy Modelling*, 42 (2), 367-384.

Colombino U. and N. Islam (2018). Basic Income and Flat Tax: The Italian scenario (with N. Islam), *CESifo Forum* 19(3), 20-29, <https://www.cesifo.org/en/publikationen/2018/article-journal/basic-income-and-flat-tax-italian-scenario>

Colombino U. (2015). Five crossroads on the way to basic income. An Italian tour, *Italian Economic Journal*, 2015, 1(3), 353-389.

Colombino U. (2013). A new equilibrium simulation procedure with discrete choice models, *International Journal of Microsimulation*, 6(3), 25-49.

Colombino, U. and E. Narazani (2013). Designing a Universal Income Support Mechanism for Italy. An Exploratory Tour, *Basic Income Studies*, 8(1), 1-17.

Creedy, J. and N. Héroult (2012). Welfare-improving income tax reforms: a microsimulation analysis. *Oxford Economic Papers*, 64(1), 128-150.

Dagsvik, J. and Strøm, S. (2006). Sectoral labour supply, choice restrictions and functional form. *Journal of Applied Econometrics* 21(6), 803–826.

Deaton, A. and J. Muellbauer (1980). *Economics and Consumer Behavior*, Cambridge University Press.

Eissa, N. and J. B. Liebman (1996). Labor Supply Response to the Earned Income Tax Credit. *The Quarterly Journal of Economics*, 111(2), 605–637.

Ericson, P. and L. Flood (2012). A Microsimulation Approach to an Optimal Swedish Income Tax, *International Journal of Microsimulation*, 2(5), 2-21.

Fortin, B., Truchon, M. & Beausejour, L. (1993). On reforming the welfare system: Workfare meets the negative income tax. *Journal of Public Economics* 51(2), 119–151.

Francesconi, M., H. Rainer and W. vanderKlaau (2009). The Effects of In-Work Benefit Reform in Britain on Couples: Theory and Evidence, *Economic Journal*, Royal Economic Society, 119(535), 66-100.

Gentilini, U., M. Grosh., J. Rigolini and R. Yemtsov (2020). *Exploring Universal Basic Income : A Guide to Navigating Concepts, Evidence, and Practices*. Washington, DC: World Bank. © World Bank. <https://openknowledge.worldbank.org/handle/10986/32677> License: CC BY 3.0 IGO.

Handler, J. F., and A. S. Babcock.(2006). The failure of workfare: Another reason for a basic income guarantee. *Basic Income Studies* 1(1), 1–22.

Heathcote, J. and H. Tsujiyama (2015). Optimal Income Taxation: Mirrlees Meets Ramsey, CEPR Discussion Papers 10380, <https://ideas.repec.org/p/cpr/ceprdp/10380.html>

Islam, N and U. Colombino (2018). The case for NIT+FT in Europe. An empirical optimal taxation exercise, *Economic Modelling* 75(C), 38-69.



King M. A. (1983). Welfare analysis of tax reforms using household data. *Journal of Public Economics*, Volume 21, Issue 2, 183-214.

Kleven, H. (2020). Sufficient Statistics Revisited. NBER Working Paper No. w27242, <https://ssrn.com/abstract=3609695>

Kolm, S. C. (1976). Inequal Inequalities I, *Journal of Economic Theory*. 12, 416-442.

Magnani, R. and L. Piccoli (2020). Universal basic income with flat tax reform in France, *Journal of Policy Modeling*, 42(2), 235-249.

McFadden, D. (1978). Modelling the Choice of Residential Location. In: A. Karlqvist, F. Snickars and J. Weibull, Eds., *Spatial Interaction Theory and Planning Models*, North Holland, 1978, pp. 75-96. P. (ed.). *Frontiers in Econometrics*, Academic Press.

Mirrlees, J. A. (1971). An exploration in the theory of optimum income taxation. *Review of Economic Studies* 38(2), 175–208.

R. A. Moffitt (2003). The Negative Income Tax and the Evolution of U.S. Welfare Policy, *Journal of Economic Perspectives*, 17(3), 119-140.

Saez, E. (2002). Optimal income transfer programs: Intensive versus extensive labor supply responses. *Quarterly Journal of Economics* 117(3), 1039–1073.

Saez, E. (2001). Using elasticities to derive optimal income tax rates. *Review of Economic Studies* 68(234), 205–229.

Sefton J. and J. Van de Ven (2009). Optimal Design of Means Tested Retirement Benefits, *Economic Journal*, 119(541), 461-481.

Standing, G. (2011). Responding to the crisis: Economic stabilization grants. *Policy & Politics* 39(1), 9–25.

Schoeni, R. F. and R. M. Blank (2000). What has Welfare Reform Accomplished? Impacts on Welfare Participation, Employment, Income, Poverty, and Family Structure, NBER Working Papers 7627.--+

Sutherland, H. and F. Figari (2013). EUROMOD: the European Union tax-benefit microsimulation model. *International Journal of Microsimulation*, 6(1) 4-26.

Tuomala, M. (2010). On optimal non-linear income taxation: numerical results revisited, *International Tax and Public Finance*, 17(3), 259-270.

Tuomala, M. (1990). *Optimal Income Tax and Redistribution*, Clarendon Press, Oxford.

Van Parijs, P. and Y. Vanderborght (2017). *Basic Income*, Cambridge (Mass), Harvard University Press.

# Appendices

## Appendix A

### Approaches to empirical optimal taxation

#### *Mirrlees*

The analytical approach pioneered by Mirrlees (1971), can be summarized as follows. It assumes a population of individuals (the “agents”) with identical preferences and different skill (or productivity)  $n$  with distribution function  $F(n)$  and probability density function  $f(n)$ . A utility function  $U(C, e)$  represents the individual preferences, where  $C$  = income and  $e$  = “effort” (or labour supply). The Government (i.e. the “principal”) solves

$$\begin{aligned}
 & \max_{T(\cdot)} \int_0^{\infty} W(U(ne_n - T(n), e_n)) f(n) dn \\
 & \text{s.t.} \\
 & \int_0^{\infty} T(n) f(n) dn \geq R \\
 & e_n = \arg \max_e u(ne - T(n), e)
 \end{aligned} \tag{9}$$

The first constraint is the public budget constraint. The second one – the so-called Incentive Compatibility Constraint – says that  $e_n$  is the effort level that maximizes the utility of the agent with productivity  $n$ .  $W(\cdot)$  is a social welfare function,  $T(\cdot)$  is a TTR that must be determined optimally,  $R$  is the average tax revenue to be collected. Mirrlees (1971) solves problem (9) with optimal control techniques. As a simple example, by assuming a quasi-linear  $U(\cdot)$  – i.e. no income effects – one can obtain:

$$\begin{aligned}
 \frac{T'(n)}{1-T'(n)} &= \left(1 + \frac{1}{\eta}\right) \left(\frac{1-F(n)}{nf(n)}\right) (1-G(n)) \\
 T_0 &= \int_0^{\infty} T(n) dF(n) - R
 \end{aligned} \tag{10}$$

where  $T'(n)$  is the marginal tax rate (MTR) applied to agents with productivity  $n$  (who have income  $ne_n$ ),  $G(n)$  is a social weight that depends on  $W(\cdot)$  and  $U(\cdot)$  and is assigned to individuals with productivity  $\geq n$  and  $\eta$  denotes the elasticity of  $e$  with respect to  $n$ .  $T_0$  is a transfer paid to individuals with no income. It is common to label  $U(\cdot)$ ,  $W(\cdot)$ ,  $F(\cdot)$ ,  $f(\cdot)$ ,  $\eta$ ,  $G(\cdot)$  and  $R$  as the “primitives” (or the basic characteristics of the economy and the social preferences of the “planner”

). For any given set of “primitives” there is a corresponding optimal TTR. The empirical applications consist of computing optimal policies using formulas such as expression (10) – or generalizations of it – with imputed or calibrated “primitives”. In Mirrlees’ original formulation,  $n$  and  $e$  are not directly observed by the Government, who is constrained to tax income  $ne_n$ . When it comes to empirical applications,  $n$  might be equated to the wage rate or imputed with a calibration procedure (e.g. Brewer et al. 2008). By assuming an explicit utility function  $U(C, e)$  and using  $e_n = \arg \max_e U(ne - T(n), e)$  one can compute the gross income  $ne_n$  and write expression

***The “sufficient statistics” approach.***

Saez (2001, 2002) presents a “sufficient statistics” approach (Chetty 2009) where expressions similar to (10) can be obtained by a “perturbation” method, i.e. working out the total effect of a marginal change of taxes and setting it equal to zero at the optimum. The solution can be expressed solely in terms of directly observed variables and non-parametrically estimable parameters (the “sufficient statistics”). As an example, in Saez (2001), under appropriate conditions, the following expression is obtained:

$$\frac{T'(z)}{1-T'(z)} = \left( \frac{1}{\eta_z} \right) \left( \frac{1-H(z)}{zh(z)} \right) (1-\Pi(z)) \quad (11)$$

where  $z$  denotes taxable income,  $h(z)$  and  $H(z)$  are the density and distribution functions,  $\Pi(z)$  is a social weight assigned to people with income greater than or equal to  $z$  and  $\eta_z$  is the elasticity of  $z$  with respect to  $(1-T'(z))$ . Expression (11) is obtained without explicit structural assumptions about preferences nor about the link between the TTR  $T(\cdot)$  and  $z$ . However, it is a “snapshot” of the optimal solution and – except for special cases – does not permit to compute directly the optimal taxes. The optimal  $z$  and its distribution (and possibly also  $\eta_z$ ) depend on the optimal tax function  $T(\cdot)$ . Therefore, in order to be able to compute the optimal taxes we must specify how  $z$ ,  $H(z)$  and  $h(z)$  depend on  $T(\cdot)$ . In other words, we must go back to Mirrlees (1971), as in Saez (2001) and Brewer et al. (2008), or introduce some ad hoc assumptions as in Saez (2002). An interesting special case concerns the top marginal tax rate. There is evidence that in the range of the highest income levels, say above  $\bar{z}$ , the term  $\frac{1-H(z)}{zh(z)}$  is approximately constant. Therefore, given an

estimate of  $\eta_z$ , the top marginal tax rate  $T'(\bar{z})$  can be computed as a function of the exogenous social welfare weight  $\Pi(\bar{z})$ . A recent paper by Kleven (2020) clarifies the limitations of the “sufficient statistics” approach and confirms that by extending it in order to overcome those limits essentially brings us back to a structural approach, i.e. one with explicit structural assumptions about preferences and the link between the  $T(\cdot)$  and  $z$ .

## Appendix B

### *The empirical model of household labour supply*

The microeconomic model has the same specification as in Islam and Colombino (2018), although it is estimated on more recent datasets and on a partially different set of countries. We model the households as agents who can choose within an opportunity set  $\Omega$  containing jobs or activities characterized by hours of work  $h$ , wage rate  $w$  and sector of market job  $s$  (wage employment or self-employment) and other characteristics (observed by the household but not by us). We define  $\mathbf{h}$  and  $\mathbf{w}$  as vectors with one element for the singles and two elements for the couples,  $\mathbf{h} = \begin{pmatrix} h_F \\ h_M \end{pmatrix}$ ,  $\mathbf{w} = \begin{pmatrix} w_F \\ w_M \end{pmatrix}$ , where the subscripts F and M refer to the female and the male partner respectively. Analogously, in the case of couples,  $\mathbf{s} = \begin{pmatrix} s_F \\ s_M \end{pmatrix}$ . The above notation assumes that each household member can work only in one sector. We write the utility function of the  $i$ -th household at a  $(\mathbf{h}, \mathbf{s}, \varepsilon)$  under TTR  $\tau$  job as follows (Coda Moscarola et al. 2020):

$$U_i(\mathbf{h}, \mathbf{s}, \varepsilon; \tau) = \mathbf{Y}_i(\mathbf{h}, \mathbf{s}; \tau)' \boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})' \boldsymbol{\lambda} + \varepsilon \quad (12)$$

where:

$\boldsymbol{\gamma}$  and  $\boldsymbol{\lambda}$  are parameters to be estimated;

$\mathbf{Y}_i(\mathbf{h}, \mathbf{s}; \tau)$  is a vector including

- $C_i(\mathbf{w}_i \mathbf{h}, I_i, \mathbf{s}; \tau)$  = household disposable income on a  $(\mathbf{h}, \mathbf{s})$  job given the tax-benefit parameters  $\tau$ ;
- the square of the household disposable income  $C_i(\mathbf{w}_i \mathbf{h}, I_i, \mathbf{s}; \tau)$  defined above;
- the product of disposable income  $C_i(\mathbf{w}_i \mathbf{h}, I_i, \mathbf{s}; \tau)$  and household size  $N$  (interaction term);

$\mathbf{L}_i(\mathbf{h})$  is a row vector including

- the leisure time (defined as the total number of available weekly hours (80) minus the hours of work  $h$ ) of the two partners (for a couple) or of the individual (for a single):  $L_{ig} = 80 - h_{ig}$ , where  $g = F, M$ .
- the square of leisure time(s),  $(L_{ig})^2$ ;
- the interaction(s) of leisure time(s) with household disposable income ( $L_{ig} \times C_i$ ), with age of the couple's partners of the single, age square and three dummy variables indicating presence of children of different age range (any age, 0-6, 7-10);

$\varepsilon$  is a random variable that measures the effect of unobserved (by the analyst) characteristics of the job-household match.

The opportunity set each individual can choose among is  $\Omega = \{(0,0), (h_1, s), (h_2, s), (h_3, s)\}$ , where  $(0,0)$  denotes a non-market "job" or activity (non-participation),  $h_1, h_2, h_3$  are values drawn from the observed distribution of hours in each hour interval 1-26 (part time), 27-52 (full time), 52-80 (extra time) and sector indicator  $s$  is equal to 1 (wage employment) or 2 (self-employment).

A  $(\mathbf{h}, \mathbf{s})$  job is "available" to household  $i$  with p.d.f.  $f_i(\mathbf{h}, \mathbf{s})$ , which we call "opportunity density".

We estimate the labour supply models of couples and singles separately. In the case of singles, we have 7 alternatives, while in the case of couples, who make joint labour-supply decision, we combine the choice alternatives of two partners, thus getting 49 alternatives.

When computing the earnings of any particular job  $(\mathbf{h}, \mathbf{s})$  we face the problem that the wage rates of sector  $s$  are observed only for those who work in sector  $s$ . Moreover, for individuals who are not working we do not observe any wage rate. To deal with this issue, we follow a two-stage procedure presented in Dagsvik and Strøm (2006) and also adopted in Coda-Moscarola et al. (2020). The procedure is analogous to the well-known Heckman correction for selectivity but is specifically appropriate for distribution assumed for  $\varepsilon$ . The random component of the impute wage is taken into account when simulating the expected likelihood

By assuming the  $\varepsilon$  is i.i.d. Type I extreme value we obtain the following expression for the probability that household  $i$  holds a  $(\mathbf{h}_i, \mathbf{s}_i)$  job (e.g. Aaberge and Colombino 2013)



$$P_i(\mathbf{h}_i, \mathbf{s}_i; \boldsymbol{\tau}) = \frac{\exp\{\mathbf{Y}_i(\mathbf{h}_i, \mathbf{s}_i; \boldsymbol{\tau})' \boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h}_i)' \boldsymbol{\lambda} + \ln f_i(\mathbf{h}_i, \mathbf{s}_i)\}}{\sum_{\mathbf{s}} \sum_{\mathbf{h} \in \Omega} \exp\{\mathbf{Y}_i(\mathbf{h}, \mathbf{s}; \boldsymbol{\tau})' \boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})' \boldsymbol{\lambda} + \ln f_i(\mathbf{h}, \mathbf{s})\}} \quad (13)$$

By choosing a convenient (uniform with peaks") specification for the opportunity density  $f(.,.)$  it turns out that expression (13) can be rewritten as follows (e.g. Aaberge and Colombino 2013, Colombino 2013),

$$P_i(\mathbf{h}_i, \mathbf{s}_i; \boldsymbol{\tau}) = \frac{\exp\{\mathbf{Y}_i(\mathbf{h}_i, \mathbf{s}_i; \boldsymbol{\tau})' \boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h}_i)' \boldsymbol{\lambda} + \mathbf{D}_i(\mathbf{h}_i, \mathbf{s}_i)' \boldsymbol{\delta}\}}{\sum_{\mathbf{s}} \sum_{\mathbf{h} \in \Omega} \exp\{\mathbf{Y}_i(\mathbf{h}, \mathbf{s}; \boldsymbol{\tau})' \boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})' \boldsymbol{\lambda} + \mathbf{D}_i(\mathbf{h}, \mathbf{s})' \boldsymbol{\delta}\}} \quad (14)$$

where, for a single household,  $\mathbf{D}_i$  is the vector (with  $1[.]$  denoting the indicator function)

$$\begin{aligned} D_{1,0} &= 1[s = 1, h > 0], \\ D_{1,1} &= 1[s = 1, 1 \leq h \leq 26], \\ D_{1,2} &= 1[s = 1, 27 \leq h \leq 52], \\ D_{2,0} &= 1[s = 2, h > 0], \\ D_{2,1} &= 1[s = 2, 1 \leq h \leq 26], \\ D_{2,2} &= 1[s = 2, 27 \leq h \leq 52]. \end{aligned} \quad (15)$$

and  $\boldsymbol{\delta}$  is vector of parameters to be estimated. For couples,  $\mathbf{D}_i$  contains two analogous sets of variables, one for each partner ( $F$  = wife,  $M$  = husband):

$$\begin{aligned}
D_{F,1,0} &= 1[s_F = 1, h_F > 0], \\
D_{F,1,1} &= 1[s_F = 1, 1 \leq h_F \leq 26], \\
D_{F,1,2} &= 1[s_F = 1, 27 \leq h_F \leq 52], \\
D_{F,2,0} &= 1[s_F = 2, h_F > 0], \\
D_{F,2,1} &= 1[s_F = 2, 1 \leq h_F \leq 26], \\
D_{F,2,2} &= 1[s_F = 2, 27 \leq h_F \leq 52] \\
D_{M,1,0} &= 1[s_M = 1, h_M > 0], \\
D_{M,1,1} &= 1[s_M = 1, 1 \leq h_M \leq 26], \\
D_{M,1,2} &= 1[s_M = 1, 27 \leq h_M \leq 52], \\
D_{M,2,0} &= 1[s_M = 2, h_M > 0], \\
D_{M,2,1} &= 1[s_M = 2, 1 \leq h_M \leq 26], \\
D_{M,2,2} &= 1[s_M = 2, 27 \leq h_M \leq 52].
\end{aligned} \tag{16}$$

It can be shown (Colombino 2013) that the coefficients  $\delta$  of the dummies  $\mathbf{D}$  depend on the number and types of available jobs and therefore reflect quantity constraints in the opportunity sets.

The model is a simplified version of the so-called RURO model (Aaberge and Colombino 2013, 2014). The main simplification concerns the wage rates. In the most general versions of the RURO model the wage rates densities are estimated simultaneously with the preference parameters and the hours' opportunity density. In this paper we use instead pre-estimated wage densities.

Expression (14) and its extensions to couples are the contribution to the likelihood function to be maximized in order to estimate the parameters  $\gamma$ ,  $\lambda$  and  $\delta$ .

The datasets used in the analysis are the EUROMOD input data based on the European Union Statistics on Income and Living Conditions (EU-SILC) for the year 2015 in France, Italy, Germany, Luxembourg, Spain and the United Kingdom. The input data provide all required information on demographic characteristics and human capital, employment and wages of household members, as well as information about various sources of non-labour income. We apply common sample selection criteria for all countries under study by selecting individuals in the age range 18-55 who are not retired or disabled. EUROMOD<sup>19</sup> is used for two different operations. First, for every

---

<sup>19</sup> EUROMOD is a large-scale pan-European tax-benefit static micro-simulation engine (e.g. Sutherland and Figari, 2013). It covers the tax-benefit schemes of the majority of European countries and allows computation of predicted household disposable income, on the basis of gross earnings, employment and other household characteristics.

household in the sample computes the net available income under the current TTR at each of the 49 (7) alternatives available to the couples (singles). The net available incomes are used in the estimation of the labour supply model. Second, for each household, it computes the gross income at each alternative. Gross incomes are used in the simulation and optimization steps, where EUROMOD is not used anymore and new values of net available incomes are generated by applying the new TTRs to the gross incomes.

The estimates of the model are reported in Tables B1 – B12.

The analytical approach typically uses point estimates of the wage elasticities of labour supply, for example according to expressions like (10) or (11) in Appendix A. As explained in the illustration of the computational approach, we do not use elasticity estimates in order to identify optimal TTRs, instead we directly use the microeconomic model to iteratively simulate decisions in order to identify optimal TTRs. We do use extensive and intensive elasticities (as reflecting “primitive” characteristics of the economy) in the exercise that “maps” from the primitives to the optimal TTRs (explained in Section 3 and commented in Section 4). Moreover, elasticities estimates are useful to illustrate how the model works. We report them in Table B13.

*Estimates of the Labour Supply Model*

Table B1 – Maximum likelihood estimates – couples (France)

Model component	Variable	Coef.	Std. Err.
Opportunity density		$\delta$	
	Employee_Man	0.4761696	0.3665576
	Self-employed_Man	0.2130577	0.3805561
	Employee_Woman	-0.3649212	0.2853927
	Self-employed_Woman	-1.426779	0.3241603
	Part-time_Employee_Man	-0.3805255	0.2414433
	Full-time_Employee_Man	2.83453	0.1249029
	Part-time_Self-employed_Man	-1.841048	0.324269
	Full-time_Self-employed_Man	0.2870089	0.1540075
	Part-time_Employee_Woman	0.6361778	0.2170085
	Full-time_Employee_Woman	2.698627	0.1676399
	Part-time_Self-employed_Woman	-1.014395	0.3149686
	Full-time_Self-employed_Woman	0.6781008	0.2277644
Y vector		$\gamma$	
	Household_Disposable_income	0.0003342	0.0001334
	Hosuhold_Disposable_income squared	1.61E-08	6.75E-09
L vector	Household_size X Household_disposable_income	-0.0000513	0.0000175
		$\lambda$	
	Leisure_Male	0.1256514	0.0281893
	Leisure_Man squared	0.0000173	0.0001373
	Leisure_Woman	0.163189	0.0255661
	Leisure_Woman squared	-0.000107	0.0001529
	Leisure_Man X Household_disp_income	-7.88E-06	1.02E-06
	Leisure_Woman X Household_disp_income	-1.54E-07	8.04E-07
	Leisure_Man X Age_Man	-0.0059183	0.0011829
	Leisure_Woman X Age_Woman	-0.0085386	0.0009848
	Leisure_Man X Age_Man squared	0.0000742	0.0000138
	Leisure_Woman X Age_Woman squared	0.0001108	0.000012
	Leisure_Man X No. Children	-0.0026133	0.0017907
	Leisure_Woman X No. Children	0.0084813	0.0015617
	Leisure_Man X No. Children0-6	0.0027957	0.0025747
	Leisure_Man X No. Children7-10	0.006507	0.0027523
	Leisure_Woman X No. Children0-6	0.006981	0.0021144
Leisure_Woman X No. Children7-10	0.0007436	0.002288	
Leisure_Woman X Leisure_Man	0.0001134	0.0000948	
Other	N. observations (N. couples*49 alternatives)	195804	
	N. couples	3996	
	LR chi2(32)	15140.15	
	Prob > chi2	0	
	Pseudo R2	0.4868	
	Log likelihood	-7981.6412	

Table B2– Maximum likelihood estimates – singles (France)

Model component	Variable	Male		Female	
		Coef.	Std. Err.	Coef.	Std. Err.
Opportunity density		$\delta$		$\delta$	
	Employee	0.199478	0.533068	-1.19744	0.470041
	Self_employed	-0.21333	0.593638	-1.68397	0.553979
	Part-time_Employee	-0.69588	0.393698	1.256628	0.352174
	Full-time_Employee	2.213382	0.253926	2.994315	0.267448
	Part-time_Self-employed	-2.70132	0.585303	-1.80785	0.627849
	Full-time_Self-employed	-0.28412	0.336337	0.368914	0.377033
Y vector		$\lambda$		$\lambda$	
	Disposable income	-0.00012	0.00024	7.55E-05	0.000379
	Disposable income squared	4.53E-08	2.07E-08	6.64E-08	4.13E-08
	Household size X Disp_income	-5.6E-05	4.62E-05	-6.7E-05	6.84E-05
L vector		$\lambda$		$\lambda$	
	Leisure	0.129227	0.029882	0.15447	0.034747
	Leisure2	-8.1E-05	0.000239	-9.2E-05	0.000254
	Leisure X Disposable income	1.01E-06	2.34E-06	6.43E-07	3.40E-06
	Leisure X Age	-0.00516	0.000963	-0.0075	0.001086
	Leisure X Age squared	6.36E-05	1.21E-05	0.000092	1.34E-05
	Leisure X No. Children	-0.01374	0.005121	0.006768	0.003433
	Leisure X No. Children 0-6	-0.00814	0.019875	0.015925	0.00544
	Leisure X No. Children 7-10	0.011728	0.010413	0.008727	0.004892
Other					
	N. observations (N. single*7 alternatives)	9331		10465	
	N. single	1333		1495	
	LR chi2(17)	2318.15		2657.35	
	Prob > chi2	0		0	
	Pseudo R2	0.4468		0.4567	
	Log likelihood	-1434.83		-1580.46	

Table B3– Maximum likelihood estimates – couples (Germany).

Model component	Variable	Coef.	Std. Err.
Opportunity density		$\delta$	
	Employee_Man	-0.5269339	0.435726
	Self-employed_Man	-1.611115	0.4373339
	Employee_Woman	-1.560942	0.2543809
	Self-employed_Woman	-3.580387	0.3637122
	Part-time_Employee_Man	-0.9591195	0.2591677
	Full-time_Employee_Man	2.516702	0.1205793
	Part-time_Self-employed_Man	-1.797576	0.3673868
	Full-time_Self-employed_Man	0.940684	0.1597094
	Part-time_Employee_Woman	1.829915	0.2318468
	Full-time_Employee_Woman	2.531309	0.1913391
	Part-time_Self-employed_Woman	0.8422771	0.341443
	Full-time_Self-employed_Woman	1.067966	0.2761494
Y vector		$\gamma$	
	Household_Disposable_income	0.001699	0.0001136
	Hosuhold_Disposable_income squared	-5.60E-08	5.58E-09
	Household_size X Household_disposable_income	0.0000479	0.0000222
L vector		$\lambda$	
	Leisure_Male	0.184064	0.0264439
	Leisure_Man squared	-0.0004684	0.0002088
	Leisure_Woman	0.2376265	0.0251642
	Leisure_Woman squared	-0.0011735	0.00014
	Leisure_Man X Household_disp_income	-0.0000138	7.25E-07
	Leisure_Woman X Household_disp_income	-9.65E-06	5.79E-07
	Leisure_Man X Age_Man	-0.0030924	0.0009326
	Leisure_Woman X Age_Woman	-0.0046903	0.0009542
	Leisure_Man X Age_Man squared	0.0000357	0.0000106
	Leisure_Woman X Age_Woman squared	0.0000713	0.0000112
	Leisure_Man X No. Children	-0.0017621	0.0019208
	Leisure_Woman X No. Children	0.0231159	0.0016417
	Leisure_Man X No. Children0-6	0.0184762	0.0025177
	Leisure_Man X No. Children7-10	-0.0007047	0.0032015
	Leisure_Woman X No. Children0-6	0.01427	0.0025455
Leisure_Woman X No. Children7-10	0.0037864	0.0026824	
Leisure_Woman X Leisure_Man	-0.0003689	0.0000744	
Other	N. observations (N. couples*49 alternatives)	201243	
	N. couples	4107	
	LR chi2(32)	14394.76	
	Prob > chi2	0	
	Pseudo R2	0.4503	
	Log likelihood	-8786.3249	

Table B4– Maximum likelihood estimates – singles (Germany)

Model component	Variable	Male		Female	
		Coef.	Std. Err.	Coef.	Std. Err.
	Opportunity density	$\delta$		$\delta$	
	Employee	-0.80193	0.57248	-1.43637	0.446197
	Self_employed	0.179376	0.597029	-8.15099	0.661612
	Part-time_Employee	-0.32109	0.406771	0.579569	0.34354
	Full-time_Employee	2.865958	0.240845	2.703306	0.249755
	Part-time_Self-employed	-2.54205	0.498362	4.688473	0.595456
	Full-time_Self-employed	0.184833	0.295293	4.011293	0.42799
Y vector		$\gamma$		$\gamma$	
	Disposable income	0.003328	0.000448	0.003246	0.000196
	Disposable income squared	-7.89E-07	7.78E-08	-1.49E-07	1.28E-08
	Household size X Disp_income	6.42E-04	1.93E-04	8.93E-05	3.80E-05
L vector		$\lambda$		$\lambda$	
	Leisure	0.241596	0.03538	0.281701	0.033209
	Leisure2	-0.00076	0.000296	-0.00141	0.000258
	Leisure X Disposable income	-4.42E-05	2.40E-06	-3.86E-05	2.08E-06
	Leisure X Age	-0.00382	0.001035	-0.00337	0.001006
	Leisure X Age squared	5.09E-05	1.24E-05	4.29E-05	1.20E-05
	Leisure X No. Children	-0.02709	0.012505	0.014031	0.003459
	Leisure X No. Children 0-6	0.038387	0.020631	0.0287	0.007638
	Leisure X No. Children 7-10	0.016741	0.021338	0.017555	0.006452
Other					
	N. observations (N. single*7 alternatives)	10,283		12,551	
	N. single	1469		1793	
	LR chi2(17)	2983.13		3608.3	
	Prob > chi2	0		0	
	Pseudo R2	0.5218		0.5171	
	Log likelihood	-1366.98		-1684.87	

Table B5 – Maximum likelihood estimates – couples (Italy)

Model component	Variable	Coef.	Std. Err.
Opportunity density		$\delta$	
	Employee_Man	-2.227042	0.3359151
	Self-employed_Man	-1.793772	0.3327547
	Employee_Woman	-4.205803	0.3711781
	Self-employed_Woman	-3.159583	0.3091701
	Part-time_Employee_Man	1.810835	0.2235256
	Full-time_Employee_Man	3.457804	0.1466732
	Part-time_Self-employed_Man	-1.142189	0.2861769
	Full-time_Self-employed_Man	1.827801	0.1352579
	Part-time_Employee_Woman	3.522802	0.3488772
	Full-time_Employee_Woman	4.233018	0.3257372
	Part-time_Self-employed_Woman	0.2200945	0.3028192
	Full-time_Self-employed_Woman	1.989132	0.2580389
Y vector		$\gamma$	
	Household_Disposable_income	0.0005129	0.0001534
	Hosuhold_Disposable_income squared	1.36E-08	7.25E-09
	Household_size X Household_disposable_income	-0.0001608	0.0000251
L vector		$\lambda$	
	Leisure_Male	0.0030689	0.05153
	Leisure_Man squared	-0.0000926	0.0001607
	Leisure_Woman	0.2598116	0.0365898
	Leisure_Woman squared	-0.000653	0.0001763
	Leisure_Man X Household_disp_income	4.38E-06	1.43E-06
	Leisure_Woman X Household_disp_income	-5.81E-07	1.01E-06
	Leisure_Man X Age_Man	-0.0015349	0.0025113
	Leisure_Woman X Age_Woman	-0.0097254	0.0016741
	Leisure_Man X Age_Man squared	0.0000135	0.0000318
	Leisure_Woman X Age_Woman squared	0.0001141	0.0000223
	Leisure_Man X No. Children	-0.0081218	0.0022336
	Leisure_Woman X No. Children	0.0078869	0.0017578
	Leisure_Man X No. Children0-6	0.0076125	0.0026554
	Leisure_Man X No. Children7-10	0.0002707	0.0028172
	Leisure_Woman X No. Children0-6	-0.0054445	0.0020634
	Leisure_Woman X No. Children7-10	-0.0009139	0.0020886
		Leisure_Woman X Leisure_Man	0.0003854
Other	N. observations (N. couples*49 alternatives)	188405	
	N. couples	3845	
	LR chi2(32)	10209.91	
	Prob > chi2	0	
	Pseudo R2	0.3411	
	Log likelihood	-9859.09	



Table B6– Maximum likelihood estimates – singles (Italy)

Model component	Variable	Male		Female	
		Coef.	Std. Err.	Coef.	Std. Err.
Opportunity density		$\delta$		$\delta$	
	Employee	-1.22117	0.331639	-3.43019	0.3787
	Self_employed	-0.47643	0.315555	-2.81075	0.350903
	Part-time_Employee	1.263827	0.268794	3.554593	0.34008
	Full-time_Employee	3.310487	0.207522	4.654217	0.303264
	Part-time_Self-employed	-2.2652	0.32631	0.618142	0.341357
	Full-time_Self-employed	1.473456	0.180946	2.786139	0.266647
Y vector		$\gamma$		$\gamma$	
	Disposable income	0.000114	0.000145	0.0003	0.000255
	Disposable income squared	5.12E-09	1.08E-08	6.55E-09	3.11E-08
	Household size X Disp_income	-5.5E-05	4.01E-05	-0.00011	4.77E-05
L vector		$\lambda$		$\lambda$	
	Leisure	0.280595	0.024332	0.312801	0.030346
	Leisure2	0.000164	0.000173	0.000428	0.000198
	Leisure X Disposable income	1.36E-06	1.55E-06	-1.91E-07	2.59E-06
	Leisure X Age	-0.01438	0.001037	-0.01841	0.001297
	Leisure X Age squared	0.000176	1.51E-05	0.000225	1.84E-05
	Leisure X No. Children	-0.0191	0.01175	0.005966	0.003381
	Leisure X No. Children 0-6	0.007813	0.020605	0.00305	0.005703
	Leisure X No. Children 7-10	0.011513	0.022161	-0.00433	0.005772
Other					
	N. observations (N. single*7 alternatives)	22190		18270	
	N. single	3170		2610	
	LR chi2(17)	4055.02		3501.41	
	Prob > chi2	0		0	
	Pseudo R2	0.3287		0.3447	
	Log likelihood	-4141.03		-3328.12	

Table B7– Maximum likelihood estimates – couples (Luxembourg)

Model component	Variable	Coef.	Std. Err.
Opportunity density		$\delta$	
	Employee_Man	2.798179	1.230943
	Self-employed_Man	1.196799	1.218041
	Employee_Woman	-1.670879	0.4877308
	Self-employed_Woman	-3.273727	0.5811094
	Part-time_Employee_Man	-0.9321119	0.5778732
	Full-time_Employee_Man	2.740097	0.2477136
	Part-time_Self-employed_Man	-3.276221	1.176261
	Full-time_Self-employed_Man	0.3923308	0.4062014
	Part-time_Employee_Woman	2.251194	0.381928
	Full-time_Employee_Woman	3.024338	0.2864887
	Part-time_Self-employed_Woman	-0.0916981	0.6417357
	Full-time_Self-employed_Woman	0.9017009	0.4806236
Y vector		$\gamma$	
	Household_Disposable_income	0.0001153	0.0001343
	Hosuhold_Disposable_income squared	-2.43E-09	2.07E-09
	Household_size $\wedge$ —Household_disposable_income	-1.63E-06	0.000023
L vector		$\lambda$	
	Leisure_Male	-0.0472945	0.0551945
	Leisure_Man squared	0.0014071	0.0004473
	Leisure_Woman	0.0416601	0.0425495
	Leisure_Woman squared	0.0003121	0.0002634
	Leisure_Man X Household_disp_income	1.64E-06	8.45E-07
	Leisure_Woman X Household_disp_income	1.18E-07	8.47E-07
	Leisure_Man X Age_Man	-0.0038039	0.0021464
	Leisure_Woman X Age_Woman	-0.0059885	0.0016256
	Leisure_Man X Age_Man squared	0.0000479	0.0000254
	Leisure_Woman X Age_Woman squared	0.0000904	0.0000201
	Leisure_Man X No. Children	-0.0067684	0.0038964
	Leisure_Woman X No. Children	0.0069002	0.0027455
	Leisure_Man X No. Children0-6	0.0085339	0.0051382
	Leisure_Man X No. Children7-10	0.002834	0.0060786
	Leisure_Woman X No. Children0-6	0.0088988	0.0034797
	Leisure_Woman X No. Children7-10	0.0022974	0.0039516
		Leisure_Woman X Leisure_Man	0.0002931
Other	N. observations (N. couples*49 alternatives)	64435	
	N. couples	1315	
	LR chi2(32)	5058.95	
	Prob > chi2	0	
	Pseudo R2	0.4943	
	Log likelihood	-2588.2705	

Table B8– Maximum likelihood estimates – singles (Luxembourg)

Model component	Variable	Male		Female	
		Coef.	Std. Err.	Coef.	Std. Err.
Opportunity density		$\delta$		$\delta$	
	Employee	3.840025	1.467676	-3.83547	0.916069
	Self-employed	3.076828	1.44261	-6.09075	1.124339
	Part-time_Employee	-1.25405	0.727917	3.214104	0.688409
	Full-time_Employee	2.760224	0.389481	3.833989	0.508304
	Part-time_Self-employed	-17.2776	699.2763	2.533271	1.111393
	Full-time_Self-employed	0.139062	0.585945	2.068577	0.878229
Y vector		$\gamma$		$\gamma$	
	Disposable income	3.53E-05	0.000416	0.00036	0.000262
	Disposable income squared	-8.96E-09	2.75E-08	-8.71E-09	9.30E-09
	Household size X Disp_income	0.000177	0.000082	-4.1E-05	5.76E-05
L vector		$\lambda$		$\lambda$	
	Leisure	0.083226	0.061447	0.222152	0.066109
	Leisure2	0.00187	0.000632	-0.00012	0.000504
	Leisure X Disposable income	2.07E-07	3.91E-06	3.42E-06	2.72E-06
	Leisure X Age	-0.0096	0.001664	-0.01311	0.00192
	Leisure X Age squared	0.000118	0.000021	0.00016	2.33E-05
	Leisure X No. Children	0.010493	0.008464	0.002518	0.005455
	Leisure X No. Children 0-6	0.006816	0.029223	0.00331	0.010569
	Leisure X No. Children 7-10	0.024772	0.029139	-0.0027	0.009811
Other					
	N. observations (N. single*7 alternatives)	4123		3640	
	N. single	589		520	
	LR chi2(17)	1157.65		951.82	
	Prob > chi2	0		0	
	Pseudo R2	0.505		4703	
	Log likelihood	-567.317		5335.965	

Table B9 – Maximum likelihood estimates – couples (Spain)

Model component	Variable	Coef.	Std. Err.
Opportunity density		$\delta$	
	Employee_Man	-0.2868366	0.2843099
	Self-employed_Man	-0.7698504	0.2931603
	Employee_Woman	-2.403139	0.252017
	Self-employed_Woman	-2.585398	0.2813223
	Part-time_Employee_Man	-0.1938884	0.1981381
	Full-time_Employee_Man	2.390778	0.1070763
	Part-time_Self-employed_Man	-1.089852	0.2490981
	Full-time_Self-employed_Man	0.9119172	0.1280802
	Part-time_Employee_Woman	1.511822	0.2167088
	Full-time_Employee_Woman	2.692898	0.1672168
	Part-time_Self-employed_Woman	-0.3884766	0.2749832
	Full-time_Self-employed_Woman	0.8462308	0.2033327
Y vector		$\gamma$	
	Household_Disposable_income	-0.0001841	0.0001271
	Hosuhold_Disposable_income squared	2.51E-08	8.12E-09
	Household_size $\hat{A}$ —Household_disposable_income	-0.0000236	0.0000156
L vector		$\lambda$	
	Leisure_Male	-0.0294838	0.0239608
	Leisure_Man squared	0.0005993	0.0001227
	Leisure_Woman	0.0991669	0.0229826
	Leisure_Woman squared	-0.0003914	0.0001435
	Leisure_Man X Household_disp_income	5.62E-06	9.76E-07
	Leisure_Woman X Household_disp_income	1.22E-06	7.46E-07
	Leisure_Man X Age_Man	-0.0022498	0.0009384
	Leisure_Woman X Age_Woman	-0.0043625	0.0007961
	Leisure_Man X Age_Man squared	0.0000224	0.0000103
	Leisure_Woman X Age_Woman squared	0.0000589	9.11E-06
	Leisure_Man X No. Children	-0.0015521	0.0013717
	Leisure_Woman X No. Children	0.0036569	0.0011625
	Leisure_Man X No. Children0-6	0.0034279	0.0019022
	Leisure_Man X No. Children7-10	-0.0022426	0.0020741
	Leisure_Woman X No. Children0-6	0.0013244	0.0016922
	Leisure_Woman X No. Children7-10	0.0020022	0.0017517
	Leisure_Woman X Leisure_Man	0.00032	0.0000585
Other	N. observations (N. couples*49 alternatives)	244755	
	N. couples	4995	
	LR chi2(32)	13049.94	
	Prob > chi2	0	
	Pseudo R2	0.3357	
	Log likelihood	-12914.672	

Table B10– Maximum likelihood estimates – singles (Spain)

Model component	Variable	Male		Female	
		Coef.	Std. Err.	Coef.	Std. Err.
Opportunity density		$\delta$		$\delta$	
	Employee	-1.30198	0.428921	-1.7268	0.428239
	Self-employed	-1.41084	0.456916	-1.59374	0.505891
	Part-time_Employee	0.607968	0.329195	1.895538	0.346785
	Full-time_Employee	2.496161	0.217392	3.236458	0.246681
	Part-time_Self-employed	-1.10093	0.419394	-0.78624	0.512791
	Full-time_Self-employed	0.757978	0.25528	1.336878	0.331166
Y vector		$\gamma$		$\gamma$	
	Disposable income	0.000391	0.000202	0.000249	0.000234
	Disposable income squared	-2.86E-08	2.19E-08	-4.02E-09	2.89E-08
	Household size X Disp_income	3.35E-05	4.41E-05	0.000166	5.63E-05
L vector		$\lambda$		$\lambda$	
	Leisure	0.091073	0.027093	0.060002	0.028202
	Leisure2	7.25E-05	0.000216	0.000661	0.000213
	Leisure X Disposable income	2.92E-08	2.03E-06	2.06E-06	2.36E-06
	Leisure X Age	-0.00465	0.000802	-0.00588	0.000853
	Leisure X Age squared	5.79E-05	9.77E-06	6.94E-05	1.02E-05
	Leisure X No. Children	0.002815	0.0077	0.005045	0.002922
	Leisure X No. Children 0-6	-0.00077	0.015847	0.004662	0.006085
	Leisure X No. Children 7-10	-0.01601	0.021244	0.00168	0.005529
Other					
	N. observations (N. single*7 alternatives)	12530		12194	
	N. single	1790		1742	
	LR chi2(17)	2335.29		2421.22	
	Prob > chi2	0		0	
	Pseudo R2	0.3352		0.3571	
	Log likelihood	-2315.53		-2179.16	

Table B11 – Maximum likelihood estimates – couples (UK)

Model component	Variable	Coef.	Std. Err.
Opportunity density		$\delta$	
	Employee_Man	0.1010326	0.3378421
	Self-employed_Man	-0.6931103	0.3433464
	Employee_Woman	-2.125809	0.2756313
	Self-employed_Woman	-2.693189	0.3039796
	Part-time_Employee_Man	-1.045434	0.2048579
	Full-time_Employee_Man	2.390656	0.1114932
	Part-time_Self-employed_Man	-1.088571	0.24735
	Full-time_Self-employed_Man	1.330872	0.1382285
	Part-time_Employee_Woman	1.85452	0.2366666
	Full-time_Employee_Woman	2.886648	0.2010856
	Part-time_Self-employed_Woman	0.3428018	0.2863139
	Full-time_Self-employed_Woman	0.6378565	0.2530663
Y vector		$\gamma$	
	Household_Disposable_income	0.0001122	0.0002534
	Hosuhold_Disposable_income squared	-3.40E-08	1.75E-08
L vector	Household_size X Household_disposable_income	0.0000397	0.000025
		$\lambda$	
	Leisure_Male	0.0426497	0.0280003
	Leisure_Man squared	0.0004884	0.0001547
	Leisure_Woman	0.1642487	0.0289328
	Leisure_Woman squared	-0.0006741	0.0001594
	Leisure_Man X Household_disp_income	9.12E-07	1.76E-06
	Leisure_Woman X Household_disp_income	-7.02E-07	1.45E-06
	Leisure_Man X Age_Man	-0.006004	0.0012687
	Leisure_Woman X Age_Woman	-0.0071774	0.0013197
	Leisure_Man X Age_Man squared	0.0000731	0.0000171
	Leisure_Woman X Age_Woman squared	0.0000904	0.0000183
	Leisure_Man X No. Children	0.0023683	0.0016979
	Leisure_Woman X No. Children	0.0169523	0.0016992
	Leisure_Man X No. Children0-6	-0.0029332	0.0017903
	Leisure_Man X No. Children7-10	0.0008209	0.001984
	Leisure_Woman X No. Children0-6	0.0142435	0.0019016
	Leisure_Woman X No. Children7-10	0.0034794	0.0020909
Leisure_Woman X Leisure_Man	0.0006758	0.0000828	
Other	N. observations (N. couples*49 alternatives)	220843	
	N. couples	4507	
	LR chi2(32)	12926.1	
	Prob > chi2	0	
	Pseudo R2	0.3685	
	Log likelihood	-11077.385	

Table B12– Maximum likelihood estimates – singles (UK)

Model component	Variable	Male		Female	
		Coef.	Std. Err.	Coef.	Std. Err.
Opportunity density		$\delta$		$\delta$	
	Employee	-0.23213	0.467104	-2.73527	0.381373
	Self_employed	-0.73375	0.479139	-3.96636	0.455567
	Part-time_Employee	-0.48835	0.315075	2.234354	0.311484
	Full-time_Employee	2.423602	0.215381	3.217563	0.257497
	Part-time_Self-employed	-1.83573	0.404768	1.149457	0.429612
	Full-time_Self-employed	0.850365	0.26129	1.745809	0.371801
Y vector		$\gamma$		$\gamma$	
	Disposable income	-8.1E-05	0.000262	0.001196	0.000448
	Disposable income squared	1.29E-08	3.04E-08	-1.33E-07	6.24E-08
	Household size X Disp_income	1.13E-05	5.03E-05	-1.9E-05	6.18E-05
L vector		$\lambda$		$\lambda$	
	Leisure	0.122021	0.027423	0.248225	0.030925
	Leisure2	0.00039	0.000239	-0.00026	0.00022
	Leisure X Disposable income	1.27E-06	2.44E-06	-9.32E-06	4.05E-06
	Leisure X Age	-0.0075	0.001001	-0.01234	0.00121
	Leisure X Age squared	0.000102	1.47E-05	0.000166	1.74E-05
	Leisure X No. Children	-0.00517	0.005611	0.015035	0.002616
	Leisure X No. Children 0-6	0.020995	0.012061	0.026352	0.003583
	Leisure X No. Children 7-10	0.013849	0.010819	0.004335	0.003571
Other					
	N. observations (N. single*7 alternatives)	13937		17549	
	N. single	1991		2507	
	LR chi2(17)	2736.7		3775.11	
	Prob > chi2	0		0	
	Pseudo R2	0.3532		0.3869	
	Log likelihood	-2505.96		-2990.84	

**Table B13. Wage elasticity**

		FR		DE		IT		LU		ES		UK	
		E	I	E	I	E	I	E	I	E	I	E	I
Direct	Wife	0.87	0.65	2.3	-0.67	4.71	0.44	1.76	-0.08	3.89	0.01	3.33	0.01
	Husband	0.21	-0.02	1.11	-0.53	0.53	0.14	0.20	0.12	1.11	0.14	1.49	0.24
	Single female	1.497	0.43	2.05	-0.64	2.66	2.35	2.20	-0.14	2.20	0.01	4.9	-0.38
	Single male	1.907	-1.85	2.66	-0.66	3.70	-2.63	2.20	0.07	2.05	0.03	4.71	0.02
	All	0.87	0.36	1.90	-0.90	2.66	-0.23	1.36	0.16	2.20	0.02	3.16	-0.17
Indirect	Wife	0.87	-0.11	4.71	-3.41	4.71	-0.22	1.76	-0.44	3.88	-0.10	3.33	-0.02
	Husband	0.20	0.57	0.53	0.40	0.53	-0.04	0.20	-0.06	1.11	0.001	1.49	0.19
E = extensive elasticity I = intensive elasticity													









